

**EUROPEAN  
MECHANICS  
SOCIETY**

## **EUROMECH Colloquium 495**

**Advances in simulation of multibody system dynamics  
18-21 February 2008, Bryansk, Russia**

### **Book of Abstracts**

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EUROMECH Colloquium 495. Advances in simulation of multibody system dynamics. Book of Abstracts, Bryansk, Russia, February 18-21, 2008, Bryansk State Technical University, 2008, 70 pages.

The book is composed of abstracts presented at the colloquium on advances in simulation of multibody system dynamics. Numerical methods in MBS (DAE, stiff equations, evaluation of Jacobians, large systems of equations); simulation of flexible/rigid MBS (comparison of absolute and relative coordinate formulations); simulation of electromechanical and mechatronic systems (multiphysics and co-simulation approaches); mathematical models of force interactions (contact, tyre, fast algorithms); simulation of rail and road vehicle dynamics; simulations of wheeled robots and walking machines, biomechanical motions of animals and humans; parallel computations in MBS; development of efficient software for simulation of MBS dynamics are under consideration.

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## **EUROMECH Colloquium 495**

The scope of the colloquium covers new developments in multibody system (MBS) dynamics: advanced methods of generation and solving equations of motion; new approaches in simulation of hybrid flexible/rigid MBS and electromechanical systems; algorithms for parallel computations and their implementations; recent applications to engineering systems (rail and road vehicles, wheeled and walking robots etc.); efficiency of multibody dynamics software. It is a major goal of this colloquium to bring together people from academic institutions and from industry and to provide a basis for discussion and exchange of new concepts and ideas to scientists from both eastern and western Europe and Asia.

### **V.V. Beletskiy**

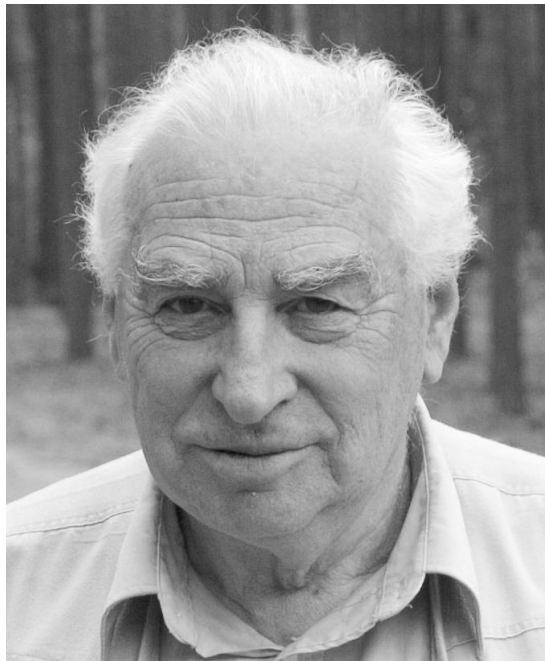
the guest of honour, Russia

#### **Professional Experience**

- 1954-present: Research Associate, Senior Research Associate, Head Research Associate, Keldysh Applied Mathematics Institute, Moscow, Russia
- 1962- present: Professor, Moscow State University, Department of Mechanics and Mathematics, part time position

#### **Principal Areas of Research**

- Space flight dynamics
- Celestial mechanics
- Robotics
- Nonlinear problems of mechanics including resonances and chaos



#### **Publications**

More than 200 papers in journals and conference proceedings including 11 monographs

#### **Additional Information**

- Member of Russian Academy of Sciences (1997)
- Member of International Astronautics Academy (1992)
- Alexander von Humboldt Prize (1992)

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## Timetable

	Monday, February 18	Tuesday, February 19	Wednesday, February 20	Thursday, February 21
7:00	Bus transfer from Moscow			
8:00		Breakfast	Breakfast	Breakfast
9:00		Session 1	Session 5	Session 7
10:00		Coffee break	Coffee break	Coffee break
11:00		Session 2	Session 6	Session 8
12:00		Lunch	Lunch	Closing
13:00		Lunch	Guided tour to Smolensk	Lunch
14:00	Registration	Session 3		Bus transfer to Moscow
15:00	Lunch	Coffee break		
16:00	Opening speech, Session 1	Session 4		
17:00				
18:00				
19:00				
20:00	Get together party	Colloquium dinner		
21:00				
22:00				

# EUROMECH Colloquium 495 Program

## Monday, February 18

- 14:00-16:00 **Registration**  
15:00-16:00 **Lunch**  
16:00-16:20 **Opening speech**

### **Euromech 495 Session 1** **Chairman: Prof. Werner Schiehlen**

- 16:20-16:50 *V.V. Beletskiy, the guest of honour (Russia)*  
Modeling of double asteroid motion  
16:50-17:10 *Peter Eberhard and Timo Gaugele (Germany)*  
Breaking of Particle Connections in Multibody Systems  
18:00-22:00 **Get together party**

## Tuesday, February 19

### **Euromech 495 Session 1** **Chairman: Prof. Werner Schiehlen**

- 08:30-08:50 *Martin Arnold (Germany)*  
Newmark Type Time Integration Methods for Constrained Mechanical Systems  
08:50-09:10 *Alexander Gorobtsov (Russia)*  
The Methods of the Transform of Multibody Systems with Redundant Constraints  
09:10-09:30 *Vladimir N. Ivanov, Vladimir A. Shimanovsky (Russia)*  
Iteration Methods of the Solution for Accelerations of the Multibody Systems  
Equations with Tree Structures  
09:30-10:00 **Coffee Break**

### **Euromech 495 Session 2** **Chairman: Prof. Javier Cuadrado**

- 10:00-10:20 *Werner Schiehlen (Germany)*  
Real-Time Simulation of Parametric Road Vehicle Models  
10:20-10:40 *Sung-Soo Kim, Wanhee Jeong, Changho Lee (Korea)*  
Real-Time Multibody Vehicle Model with Bush Compliance Effect Using Quasi-static Analysis for HILS  
10:40-11:00 *Jorge Ambrosio, Paulo Verissimo (Portugal)*  
Improved Bushing Models for Vehicle Dynamics  
11:00-11:20 *Bum S. Kim, Bong S. Kim, Hong H. Yoo (Korea)*  
Analytical Method to Analyze Tolerance Effects on Vehicle Ride Comfort  
12:00-13:00 **Lunch**



**Euromech 495 Session 3**  
**Chairman: Prof. Evtim Zahariev**

- 14:00-14:20 **Wojciech Blajer, Krzysztof Kołodziejczyk (Poland)**  
Extended DAE Formulation for Inverse Simulation of Cranes Executing Prescribed Load Motion
- 14:20-14:40 **Felix L. Chernousko, Nikolai N. Bolotnik, and Tatiana Yu. Figurina (Russia)**  
Modeling and Control of Vibration-Driven Mobile Systems with Internal Masses
- 14:40-15:00 **Tae-Won Park, Sung-Pil Jung, Kab-Jin Jun, Won-Seon Jeong (Korea)**  
Optimum Design of a Mechanism using a Multibody Model and Response Surface Analysis
- 15:00-15:20 **Yu.F. Golubev, V.V. Korianov (Russia)**  
Computer Simulation of the Six Legged Robot's 3D-Dynamics into the Media with Obstacles
- 15:20-15:40 **V.E. Pavlovsky, V.V. Evgrafov (Russia)**  
Simulation of Motion of Rover with Elastic Wheels
- 15:40-16:10 **Coffee Break**

**Euromech 495 Session 4**  
**Chairman: Prof. Jorge Ambrosio**

- 16:10-16:30 **Wan-Suk Yoo, Hyun-Woo Kim, Jeong-Han Lee, Jae-Cheol Ryu, Bo-Sun Chung, Kyung-Hun Rho (Korea)**  
Damping Models for Multibody Dynamic Simulations
- 16:30-16:50 **Ivan I. Kosenko, Evgeniy B. Alexandrov (Russia)**  
Implementation of the Hertz Contact Model on Modelica Language
- 16:50-17:10 **Andreas Müller (Germany)**  
Hysteresis Modeling in Electro-Magneto-Mechanical Systems
- 17:10-17:30 **Natalia Dokukova, Katerina Kaftaikina (Belarus)**  
Generalities of Passive Vibration Dampers Isolating Vibrations
- 18:00-22:00 **Colloquium dinner**

**Wednesday, February 20**

**Euromech 495 Session 5**  
**Chairman: Prof. Peter Eberhard**

- 08:30-08:50 **U. Lugijs, J. Cuadrado (Spain)**  
Improved Efficiency in FFR Methods for Flexible Multibody Dynamics by mean of Shape Integrals Preprocessing
- 08:50-09:10 **Evtim V. Zahariev (Bulgaria)**  
Spatial Frictional Impact of Rigid and Flexible Multibody Systems
- 09:10-09:30 **Oleg Dmitrochenko, Aki Mikkola (Finland)**  
A Formal Procedure and Invariants of a Transition from Conventional Finite Elements to the Absolute Nodal Coordinate Formulation
- 09:30-09:50 **Marko K. Matikainen (Finland), A. L. Schwab (The Netherlands)**  
Comparison of Two Thick Plate-Elements Based on the Absolute Nodal Coordinate Formulation
- 09:50-10:10 **Coffee Break**

**Euromech 495 Session 6**  
**Chairman: Prof. Ivan Kosenko**

- 10:10-10:30 *Nikolay Lysikov, Roman Kovalev, Gennadiy Mikheev (Russia)*  
Stress Load and Durability Analysis Using Multibody Approach
- 10:30-10:50 *V. Boykov (Russia)*  
Application Experience of EULER Program Complex for the Automated  
Dynamic Analysis of Multicomponent Mechanical System
- 10:50-11:10 *Thomas Grund (Germany)*  
Customized Simulators: Software Structure and Case Study
- 11:45-12:30 **Lunch**
- 12:30-22:00 **Guided tour to Smolensk**

**Thursday, February 21**

**Euromech 495 Session 7**  
**Chairman: Prof. Wojciech Blajer**

- 08:30-08:50 *V. Boykov, A. Gorobtsov, D. Pogorelov (Russia)*  
Benchmarks for Testing MBS Software Efficiency
- 08:50-09:10 *Andrey B. Byachkov, Vladimir N. Ivanov (Russia)*  
Classification of Various Forms of Dynamic Multibody Equations
- 09:10-09:30 *A.N. Tuganov (Russia)*  
A Linear Formulation for Multibody Dynamics Simulation with Unilateral  
Constraints and Coulomb Friction
- 09:30-09:50 *Paulo Flores (Portugal)*  
Dynamic Modeling and Analysis of the Human Knee Joint
- 09:50-10:10 *Al Nazer, R., Kłodowski, A., Mikkola, A., Rantalainen, T., Heinonen, A.,  
Sievänen, H. (Finland)*  
Lower Body Musculoskeletal Model with a Flexible Tibia Used in the Strain  
Analysis During Walking
- 10:10-10:40 **Coffee Break**

**Euromech 495 Session 8**  
**Chairman: Prof. Dmitry Pogorelov**

- 10:40-11:00 *Alexandre A. Zarifian, Pavel G. Kolpahchyan (Russia)*  
Computer Modeling of Electric Locomotive as Controlled Electromechanical  
System
- 11:00-11:20 *D.Yu. Pogorelov, V.A. Simonov (Russia)*  
Computer-Aided Models of Freight Cars and Their Applications to Analysis of  
Some Dynamic Problems
- 11:20-11:40 *Goryacheva I.G., Soshenkov S.N., Zakharov S.M., Zharov I.A., Yazykov V.N.  
(Russia)*  
Wear of Wheel and Rail in the Process of Vehicle-Track Interaction
- 11:40-12:00 *D. Marinova (Bulgaria)*  
Modeling and Simulation of the Dynamics of Flexible Composites
- 12:00-12:20 **Closing**
- 12:20-13:20 **Lunch**
- 14:00-22:00 **Bus transfer to Moscow**

# Lower body musculoskeletal model with a flexible tibia used in the strain analysis during walking

Al Nazer, R.<sup>1</sup>, Klodowski, A.<sup>1</sup>, Mikkola, A.<sup>1</sup>, Rantalainen, T.<sup>1,2</sup>, Heinonen, A.<sup>3</sup>, & Sievänen, H.<sup>4</sup>

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**Keywords:** Tibia, flexible multibody dynamics, loading, strain, walking.

## Abstract

The objective of this study was to show that the bone strains occur due to dynamic loading during an exercise can be analysed using the flexible multibody approach. Strains within the bone tissue play a major role in bone (re)modeling [1]. These small strains can be assessed using experimental strain gage measurements, which are challenging and invasive [2]. Further, the strain measurements are, in practise, limited to certain regions of superficial bones only, such as the anterior surface of the tibia. In this study, tibial strains occurring during walking were estimated using a numerical approach based on flexible multibody dynamics. In the introduced approach, a lower body musculoskeletal model with a flexible tibia was developed as shown in Fig 1. Inverse dynamics simulation based on the motion capture data obtained from walking at a constant velocity was used to teach the muscles in order to reproduce the motion in the forward dynamics simulation. The maximum and minimum tibial principal strains predicted by the model were 490 and -588 microstrain, respectively, which are in line with literature values from *in vivo* measurements [2], [3], [4], [5]. In conclusion, the non-invasive flexible multibody simulation approach may be used as a surrogate for experimental bone strain measurements and thus be of use in detailed strain estimations of bones in different applications. According to the authors' knowledge, this was the first time the flexible multibody approach was used in this kind of application.

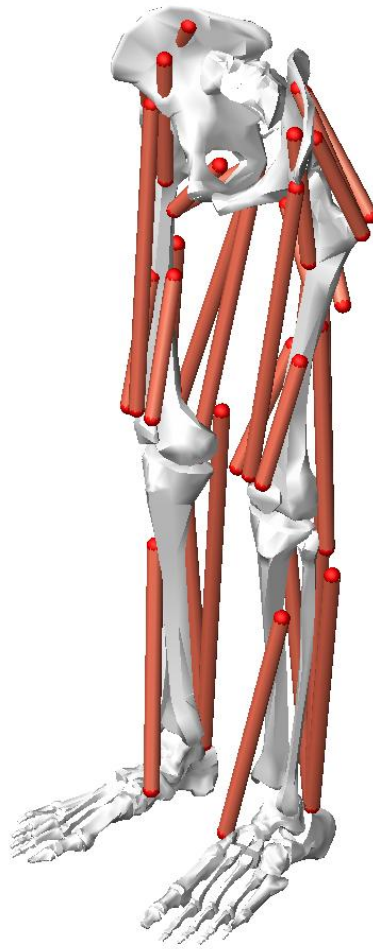


Fig 1. Lower body musculoskeletal model with a flexible tibia used in this study.

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- [2] Lanyon, L. E., Hampson, W. G., Goodship, A. E., Shah, J. S., "Bone Deformation Recorded in vivo from Strain Gauges Attached to the Human Tibial Shaft", *Acta Orthopaedica Scandinavica*, Vol. (46), 256-268, (1975).
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- [5] Burr, D. B., Milgrom, C., Fyhrie, D., Forwood, M., Nyska, M., Finestone, A., Hoshaw, S., Saiag, E., Simkin, A., "In vivo Measurement of Human Tibial Strains during Vigorous Activity", *Bone*, Vol. (18), 405-410, (1996).

# Improved Bushing Models for Vehicle Dynamics

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**Keywords:** Multibody Dynamics, Vehicle Suspensions, Elastometers, Joint Modeling.

## 1. Introduction

The development and computational implementation, on a multibody dynamics environment, of a constitutive relation to model bushing elements associated with mechanical joints used in the models of road and rail vehicles is presented here. This kind of elements can be found in a wide number of mechanisms, such as car suspensions, where they play important roles in the elimination of vibrations due to road irregularities, to allow small misalignment of axes, to reduce noise from the transmission and to decrease wear of the mechanical joints. In vehicle dynamics the ride and handling of a vehicle are conditioned by the performance of these bushing elements. Therefore, suitable bushing models for vehicle multibody models must be accurate and, at the same time, computationally efficient so that they can be included in the vehicle models and, therefore, leading to a more reliable dynamic response is obtained from the multibody simulations. Bushings are made of a special rubber, used generally in energy dissipation, which presents a nonlinear and viscoelastic relationship between the forces and moments and their corresponding displacements and rotations. In the methodology proposed here a finite element model of the bushing is developed in the framework of the finite element (FE) code ABAQUS to obtain the curves of displacement/rotation versus force/moment for different loading cases. The bushing is modeled in a multibody code as an arrangement of springs that restrain the motion between the bodies connected. The spring stiffness is obtained in the FE code. The basic ingredients of the multibody model are the same vectors and points relations used to define kinematic constraints in any multibody formulation. Four types of bushing joints are implemented: spherical, revolution, cylindrical and translational. Finally, the methodology is demonstrated through the simulation of two multibody models of a road vehicle, one with perfect kinematic joints, for the suspension sub-systems, and the other with bushing joints. The tests conducted to measure and compare the handling of the two vehicle models are include obstacle avoidance maneuver and the vehicle riding over bumps, both at several speeds.

## 2. Bushing Model for Kinematic Joints

Bushing elements reduce the wear of the components of a mechanism by using their energy absorption capacity to reduce transmitted vibrations. The bushings prevent misalignment of axes and noise reduction from the transmission. This type of elements has a large spectrum of application, although in this study, only the use of bushings in vehicle suspensions is focused. The bushing element presented in Fig. 1 is composed by elastomer and two steel sleeves, the inner and outer ring. The elastomeric material is common to all types of bushing elements.

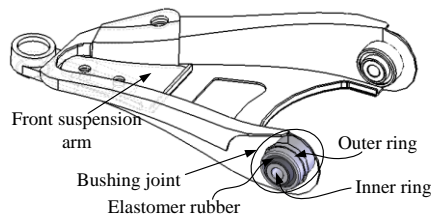


Fig. 1. Busing Joints in the arm of a McPherson suspension system

The spherical joint allows for three rotations between the bodies connected, constraining only the relative translation displacements, as presented in Fig. 2(a). In a perfect cylindrical joint the degrees of freedom to be restrained by this joint are the normal translational displacement, and the angular displacement due to the misalignment of the vectors, as presented in Fig. 2(b). For bushing joints all these restraints are relaxed being the relative displacements penalized according to a constitutive relation for the bushing material and geometry.

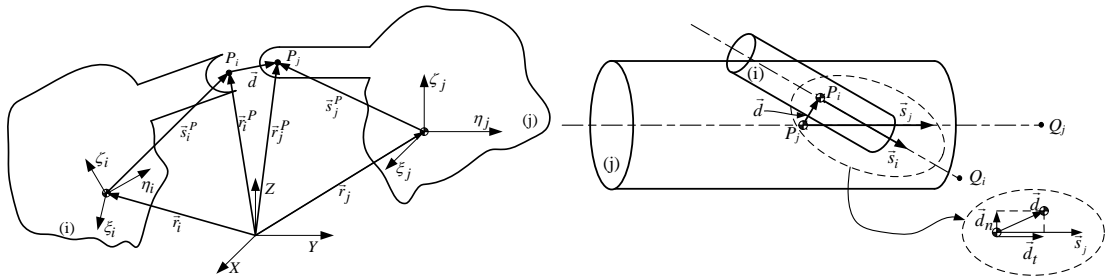


Fig. 2. Busing Joints models: (a) Spherical joint bushing; (b) Revolute joint bushing

In this study a finite element code is used for the determination of the bushing element stiffness functions. For this purpose four test cases are conducted in the FE program ABAQUS with a FE model of the bushing element. The bushing is modeled with 1404 solid "hybrid" elements denominated by C3D8H, as depicted in Fig. 3.

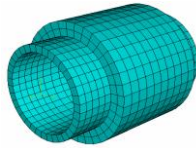


Fig. 3. Rubber bushing element of a vehicle suspension

The models for the bushing joints developed in this work are applied to a vehicle model of a small family car. Several scenarios are used to demonstrate the performance of the new methodology, among which the vehicle rides over ten bumps, both with 0,1m height, as seen in Fig. 4. This simulation excites the roll motion of the vehicle chassis, making possible an evaluation of the suspension efficiency to reduce this chassis motion. These scenarios are evaluated at three vehicle forward speeds: 60, 90 and 120Km/h.

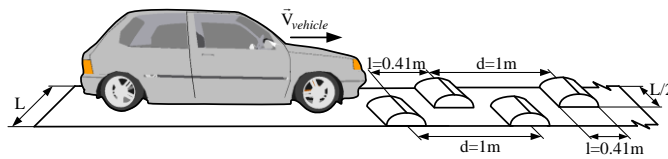


Fig. 4. Vehicle model riding over several bumps

In order to characterize the dynamic behavior of the vehicle models in this scenario, the vertical position, vertical and roll accelerations are extracted during the simulations. Those values are then compared to understand the influence of the elastic joints in the dynamic behavior of the vehicle when subjected to the scenario described.

### 3. Conclusions

It is shown that by using a reverse engineering process it was possible to build two multibody models of the vehicle, one with perfect kinematic joints and other with bushing joints. The bushing joints constitutive functions are nonlinear and their characteristics are identified using a nonlinear finite element code and are reported by the methodology developed and presented here. In the process all formulations used to characterize the bushing joints use exactly the same kinematic quantities required to characterize the perfect joints leading to a relatively unified representation of both type of joints. Finally it is shown that the computational efficiency of the vehicle models using bushing joints is superior to that of vehicle models using perfect kinematic joints.

# Newmark type time integration methods for constrained mechanical systems

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**Keywords:** Time integration, Newmark method, linearly-implicit time integration.

## 1. Introduction

The efficient numerical solution of the equations of motion for constrained mechanical systems is a classical topic in multibody dynamics. Higher order time integration methods of Runge-Kutta or BDF type with stepsize and order control are considered to define the state of the art [1]. On the other hand, Newmark type time integration methods that are well known from structural mechanics [2] may also be extended to constrained mechanical systems [3, 4] and found recently much interest in the numerical analysis community [5, 6, 7]. In the present paper, different approaches of Newmark type are compared with classical DAE time integration methods in the field of multibody dynamics.

## 2. Newmark type time integration methods

Newmark type methods are frequently used for the time integration of finite element models in structural dynamics that are represented by second order ordinary differential equations (ODEs) of large dimension. For practical application, Newmark's classical approach was extended by several authors resulting finally in the class of generalized- $\alpha$  methods [2] that will be considered throughout the present paper.

Cardona and Géradin proposed already in 1989 the extension of Newmark type methods to constrained mechanical systems applying the method to the index-3 DAE formulation of the equations of motion [3]. However, in multibody numerics, methods for first order DAEs found more interest because of its straightforward extension to the model equations of mechatronic systems that are combined of second order DAEs and first order ODEs [1, 4].

Recently, generalized- $\alpha$  methods were generalized to coupled second and first order systems [4] and are now also applied successfully for large scale simulations in multibody dynamics [5, 6]. Lunk and Simeon [5] and Jay and Negrut [6] propose the application to the stabilized index-2 DAE formulation [1] of the equations of motion. A convergence proof for the original approach of Cardona and Géradin (index-3 formulation) is given by Arnold and Bröls [7].

In the index-3 case, the convergence analysis is based on an equivalent multistep representation of generalized- $\alpha$  methods. In the present paper, this technique is extended to the (stabilized) index-2 DAE formulation resulting in improved error estimates for the Lagrangian multipliers. Second order convergence is demonstrated for all solution components including the algebraic ones.

Numerical tests for a benchmark problem compare both approaches and illustrate practical implications of the convergence analysis. Furthermore, the generalized- $\alpha$  method is implemented in a developer version of an industrial simulation package enabling its use in large scale industrial applications.

Finally, a linearly-implicit modification of the generalized- $\alpha$  method will be considered that avoids the corrector iteration of implicit methods which is of particular interest in real-time applications, see also [8] for a similar approach in the ODE case. Again, the results of the theoretical analysis are illustrated by simulation results for a benchmark problem.

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# Extended DAE formulation for inverse simulation of cranes executing prescribed load motion

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**Keywords:** crane dynamic and control, inverse simulation, servo constraints, differential flatness.

## 1. Introduction

Cranes belong to underactuated mechanical systems which have fewer control inputs than the degrees of freedom. The common performance goal is then a desired load trajectory, and the motion is always specified by as many desired output signals as the number of control inputs [1]. A solution to the inverse simulation problem, in which the control inputs of the underactuated system required to execute the partly specified motion are determined, is possible due to the problem is *differentially flat* [2], [3], and *flatness* means that for a given (underactuated) dynamical system a set of *flat outputs* can be found, equal in number to the number of inputs, so that all the states and inputs can be algebraically expressed in terms of the outputs and the time derivatives of the outputs up to a certain order. This provides a basis for synthesis of control laws for sufficiently smooth output functions. The flatness-based control of cranes requires very complex mathematical modeling, however, and the fourth order time derivatives of the specified outputs need to be used [4]. The alternative DAE formulation proposed in [5] is much more straightforward and applicable, involving only the third order time derivatives of the outputs. While in [5] the crane dynamics was defined in independent coordinates, in the present formulation dependent coordinates are used, which involves constraints on the system. The specified control outputs are then expressed in terms of the dependent coordinates, which leads to *servo constraints* on the controlled system, in addition to the previously mentioned *passive constraints* in the classical sense. The realization of passive and servo constraints, respectively, by passive constraint reactions and control forces, is qualitatively different, and for differentially flat underactuated systems some servo constraints are always realized by control forces which are tangent to the constraint manifolds. This implies that the initial governing equations of the inverse simulation problem are formulated as index five DAEs, which are then transformed to an equivalent index three DAEs. An effective method for solving the DAEs, proposed in [5], is then used to solve the DAEs. The feedforward control law obtained this way is then extended by a closed-loop control strategy with feedback of the actual errors to provide stable tracking of the required reference load trajectories in the presence of perturbations.

## 2. Modeling and simulation framework

The crane is modeled as an  $f$ -degree-of-freedom system with  $m$  control inputs  $\mathbf{u} = [u_1 \ \dots \ u_m]^T$ ,  $m < f$ . As different to the formulation provided in [5] where  $f$  independent coordinates are used, the crane dynamics is described here in  $n = 2m > f$  non-minimal coordinates  $\mathbf{q} = [\mathbf{p}^T \ \mathbf{y}^T]^T$ , involving  $m$  (actuated) robot coordinates  $\mathbf{p}$  and  $m$  load coordinates  $\mathbf{y}$  (a similar formulation of crane dynamics is provided in [4]). There are then  $p = n - f$  constraints on the system described in the dependent coordinates  $\mathbf{q}$ , due to the cable longitudinal stiffness,  $\Phi_p(\mathbf{q}) = \mathbf{0}$ , called *passive constraints* hereafter ( $p = 1$  for the load modeled as mass point suspended by a single cable). The crane dynamic equations in  $\mathbf{q}$  are then  $\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} - \mathbf{B}^T\mathbf{u} - \mathbf{C}_p^T\boldsymbol{\lambda}$ , where  $\mathbf{M}$  is the  $n \times n$  generalized mass matrix, the  $n$ -vector  $\mathbf{f}$  contains the applied forces and dynamic terms,  $\mathbf{B}^T$  is the  $n \times m$  control input matrix,  $\mathbf{C}_p = \partial\Phi_p / \partial\mathbf{q}$  is the  $p \times n$  contact constraint matrix, and the  $p$  Lagrange multipliers  $\boldsymbol{\lambda}$  are the cable forces. Compared to the formulation in independent coordinates [5], the crane dynamic equations in  $\mathbf{q}$  are advantageous due to the ease of derivation and simplicity of structure. i.e.  $\mathbf{M}$  is a (block) diagonal and constant matrix, and the dynamic equations related to  $\mathbf{p}$  and  $\mathbf{y}$  are coupled only through the generalized constraint force  $\mathbf{f}_p = -\mathbf{C}_p^T\boldsymbol{\lambda}$ . Simplifications relate also to the formulation of  $m$  *servo constraints*, whose equations are simply  $\Phi_s(\mathbf{q}, t) \equiv \mathbf{y} - \mathbf{y}_d(t) = \mathbf{0}$ , where  $\mathbf{y}_d(t)$  are the specified outputs (load coordinates).

As mentioned, the realization of passive and servo constraints is substantially different, and a specific methodology must be applied to solve the inverse simulation problem: *given the desired load motion  $\mathbf{y}_d(t)$ , determine the control inputs  $\mathbf{u}_d(t)$  that force the crane to complete the prescribed motion*. In short, both  $p$  passive and  $m$  servo constraints,  $\Phi(\mathbf{q}, t) \equiv [\Phi_p^T \ \Phi_s^T]^T = \mathbf{0}$ , are twice differentiated with respect to time to obtain

$\mathbf{C}(\mathbf{q})\ddot{\mathbf{q}} - \xi(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{0}$ , where  $\mathbf{C} = [\mathbf{C}_p^T; \mathbf{C}_s^T]^T$  is the  $(p+m) \times n$  matrix with  $\mathbf{C}_p = \partial\Phi_p / \partial\mathbf{q}$  and  $\mathbf{C}_s = \partial\Phi_s / \partial\mathbf{q}$ , and the  $(p+m)$ -vector  $\xi = [\xi_p^T; \xi_s^T]^T$  is formed by  $\xi_p = -\mathbf{C}_p \dot{\mathbf{q}}$  and  $\xi_s = \dot{\mathbf{y}}_d - \mathbf{C}_s \dot{\mathbf{q}}$ . In the  $n$ -space related to  $\dot{\mathbf{q}}$ ,  $\mathbf{C}$  defines the  $(p+m)$ -dimensional constrained and specified subspace, and the complementing  $k$ -dimensional ( $k = n - p - m$ ) subspace where the motion is neither constrained nor specified is then defined by an  $n \times k$  matrix  $\mathbf{D}$  chosen so that  $\mathbf{D}^T \mathbf{C}^T = \mathbf{0}$ , and as such  $\mathbf{D}^T \mathbf{C}_p^T = \mathbf{0}$  and  $\mathbf{D}^T \mathbf{C}_s^T = \mathbf{0}$ . After projecting the crane dynamic equations into the two subspaces, the governing equations for the inverse simulation problem are obtained as the following  $n + k + p + m + p + m = 2n + p + m$  DAEs in the same number of variables  $\mathbf{q}$ ,  $\mathbf{v}$ ,  $\lambda$  and  $\mathbf{u}$ ,

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{v} & \dot{\mathbf{q}} &= \mathbf{v} \\ \mathbf{D}^T \mathbf{M} \dot{\mathbf{v}} &= \mathbf{D}^T \mathbf{f} - \mathbf{D}^T \mathbf{B}^T \mathbf{u} & \mathbf{H}(\mathbf{q}) \dot{\mathbf{v}} &= \mathbf{h}(\mathbf{q}, \mathbf{v}, \mathbf{u}, t) \\ \mathbf{0} &= \mathbf{C} \mathbf{M}^{-1} \mathbf{f} - \mathbf{C} \mathbf{M}^{-1} \mathbf{B}^T \mathbf{u} - \mathbf{C} \mathbf{M}^{-1} \mathbf{C}_p^T \lambda - \xi & \Leftrightarrow & \mathbf{0} = \mathbf{b}(\mathbf{q}, \mathbf{v}, \mathbf{u}, \lambda, t) \\ \mathbf{0} &= \Phi(\mathbf{q}, t) & & \mathbf{0} = \Phi(\mathbf{q}, t) \end{aligned} \quad (1)$$

The solution to DAEs (1) are time-variations of the dependent state variables for the crane executing the load prescribed motion,  $\mathbf{q}_d(t) = [\mathbf{p}_d^T(t) \ \mathbf{y}_d^T(t)]^T$  and  $\mathbf{v}_d(t) = [\dot{\mathbf{p}}_d^T(t) \ \dot{\mathbf{y}}_d^T(t)]^T$ , the cable forces  $\lambda_d(t)$ , and the control  $\mathbf{u}_d(t)$  that ensures the motion realization.

The background for the described formulation/methodology, an effective code for the solution of the above DAEs, and the results of numerical simulations for the crane maneuvering the load along a specified trajectory will be the subject of the presentation. A closed-loop control strategy with feedback of the actual errors in load position to provide stable tracking of the required reference load trajectories in presence of perturbations and modeling inconsistencies will also be reported.

### 3. Further comments and conclusions

Compared to the initial governing equations of the considered inverse simulation problem, formed by  $\dot{\mathbf{q}} = \mathbf{v}$ ,  $\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} - \mathbf{B}^T \mathbf{u} - \mathbf{C}_p^T \lambda$  and  $\Phi(\mathbf{q}, t) = \mathbf{0}$ , which are DAEs of index five [5] dependent on  $\mathbf{y}_d$ , the present DAEs (1) are of index three and are depend on the outputs and their time derivatives up to the second order,  $\ddot{\mathbf{y}}_d$ , which is consequent to twice differentiation with respect to time of the servo constraint equations. As shown in [4] and theoretically motivated in [2] and [3], further twice differentiation with respect to time and (complex) manipulations of the governing equations result in explicit resolution of  $\mathbf{q}$ ,  $\mathbf{v}$ ,  $\lambda$  and  $\mathbf{u}$  in terms of the outputs and their time derivatives up to the fourth order. More strictly, the following algebraic relations can (theoretically) be found:  $\mathbf{q} = \mathbf{q}(t, \mathbf{y}_d, \dot{\mathbf{y}}_d, \ddot{\mathbf{y}}_d)$ ,  $\lambda = \lambda(t, \mathbf{y}_d, \dot{\mathbf{y}}_d, \ddot{\mathbf{y}}_d)$ ,  $\mathbf{v} = \mathbf{v}(t, \mathbf{y}_d, \dot{\mathbf{y}}_d, \ddot{\mathbf{y}}_d, \mathbf{y}_d^{(3)})$  and  $\mathbf{u} = \mathbf{u}(t, \mathbf{y}_d, \dot{\mathbf{y}}_d, \ddot{\mathbf{y}}_d, \mathbf{y}_d^{(3)}, \mathbf{y}_d^{(4)})$ , which is consequent to differential flatness of the inverse simulation problem. The solution can also be interpreted as a specific index-one DAE formulation of the inverse simulation problem, allowing immediately for the dynamic analysis and synthesis of control of cranes executing load prescribed motions. Complexity of the involved mathematical modeling may, however, be discouraging. The approach motivated in this contribution is much more straightforward and applicable, and it can be ‘placed’ just ‘between’ the initial index-five DAE formulation and the resolved flatness-based (index-one DAE) formulation. Compared to the previous proposition [5], where independent coordinates were used, the present application of dependent coordinates  $\mathbf{q} = [\mathbf{p}^T \ \mathbf{y}^T]^T$  yields some modeling privileges and leads to the extended DAE formulation in which the underactuated system is subject to both passive and servo constraints.

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# Benchmarks for Testing MBS Software Efficiency

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**Keywords:** multibody system simulation, software, benchmarks.

## 1. Introduction

Benchmarking is an important tool for comparison of different multibody formalisms, numeric methods and their implementation in MBS software codes. The known test examples [1] together with publication of MBS algorithms in this handbook by developers of software exert considerable influence on the process of perfection of simulation methods. Nowadays the performance of both the hardware and software is increased, which requires more complex test models.

The test models below are proposed by developers of three Russian programs: Euler [2], FRUND [3] and Universal Mechanism [4].

## 2. Model description

Two types of test models are considered. The first one is a chain, Fig. 1, the model is intended for test of simulation of MBS with long kinematic chains without kinematic loops. The second model is a square grid, Fig. 2. It is an example of a system with large number of closed loops.

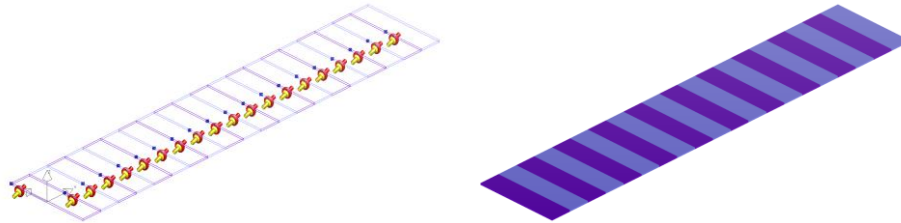


Fig. 1. Chain with 20 bodies, Euler software design

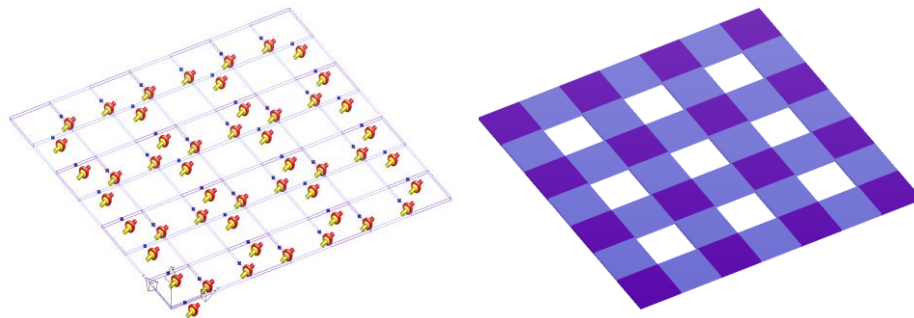


Fig. 2. 7x7 grid, Euler software design

The chain in initial state is situated along X axis. It is 5m length, 1m width, 0.02m thickness and 50 kg mass. Bodies included in the chain are homogeneous parallelepipeds. Number of bodies for different variants of simulation is 50, 100, 150, 200. Ideal spherical joints connecting pairs of bodies are located in centers of parallelepiped sides. The first body is connected to the ground by spherical joint located in the center of vertical edge (Fig. 1).

The grid consists of equal square homogeneous bodies. The grid structure contains interstices in the center of each group of 9 bodies, which makes the system statically determinate. The grid has 1m length and width, 0.01m thickness and 50 kg mass. Number of bodies for different variants of simulation is 7, 9, 11, 13 for a side, i.e. totally 49, 81, 121 and 169 bodies. Pairs of bodies are connected by ideal spherical joints, located in the centers of parallelepiped sides. The first body is connected to the ground by a spherical joint with coordinates (0.05, 0, 0).

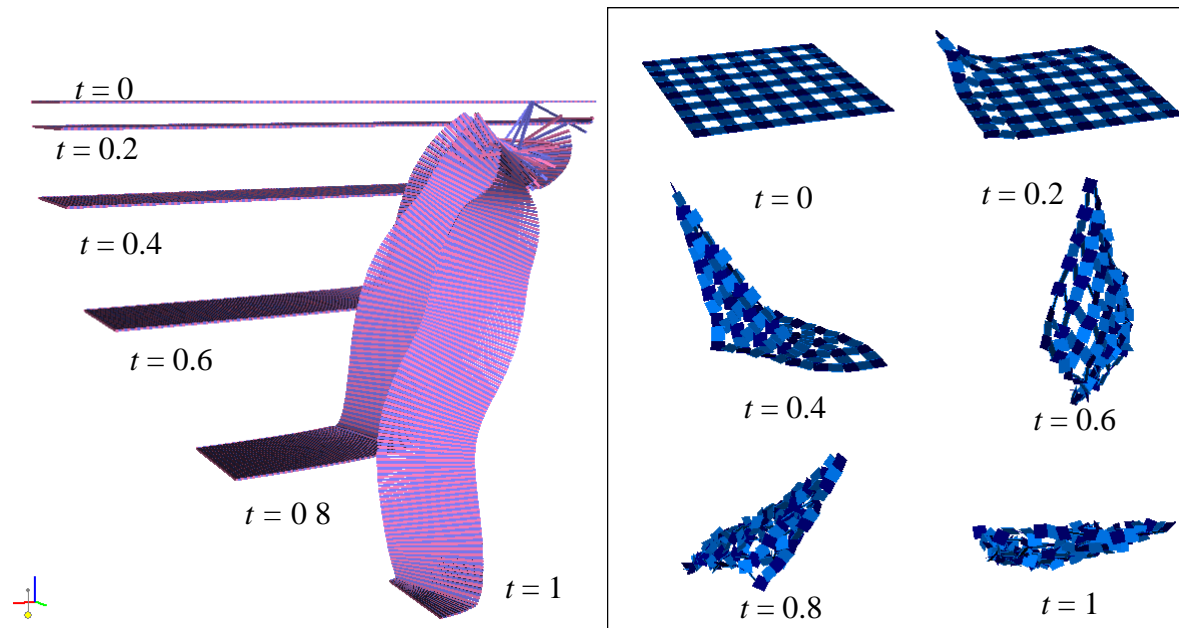


Fig. 3. Simulation of chain with 200 bodies and 13x13 grid dynamics (Universal Mechanism software)

For all cases the gravity is directed upwards along Z axis, gravitational acceleration is  $9.81 \text{ m/s}^2$ . Simulation time is 1s. Test simulations were carried-out and compared for programs [2]-[4], which use different solvers for DAE and formalisms for generation of equations.

For test of solvers in case of stiff DAE, modifications of the models above is proposed by adding linear elastic and damping torques in joints with different values of stiffness and damping constants.

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- [3] <http://frund.vstu.ru/en/frund.htm>
- [4] <http://www.umlab.ru>

# Application Experience of EULER Program Complex for the Automated Dynamic Analysis of Multicomponent Mechanical Systems

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**Keywords:** multibody system simulation, software.

EULER software system is developed to perform mathematical simulation of multi-component mechanical systems dynamics in 3D-space. EULER software can be applied in automotive industry, aircraft and rocket engineering, defense industry, robotics, machine-tool industry and others.

EULER is successfully used at enterprises from various areas of engineering, including: CAHI Central Aero-Hydrodynamics Institute, TSNIMASH Central Research Institute for Machine Building of Russian Space Agency, KHRUNICHEV State Research and Production Space Center, Research and Production Center of mechanical Engineering, TUPOLEV Central Design Bureau, KBP Instrument Design Bureau, State Research and Production Enterprise «Splav» and many other organizations.

Fig. 1 is a screenshot of EULER environment during simulation of Rockot carrier rocket lift-off. The simulation was performed by KHRUNICHEV State Research and Production Space Center. Lift-off model includes dynamics of the launcher tower+ launcher base + transporter-launcher barrel(TLB) + carrier-rocket mechanical system.

The model features:

- Flexible carrier-rocket;
- Flexible transporter-launcher barrel;
- Flexible launcher tower;
- Flexible connections carrier-rocket – TLB and TBL – carrier-rocket;
- Non-stationary wind loads on carrier-rocket TLB and launcher tower;
- Effect of transversal oscillations caused by the wind;
- Effect of TBL heating at lift-off.

Shock-free conditions of Rockot carrier-rocket lift-off were found by statistical analysis that involved stochastic variations of the carrier-rocket and launcher parameters, environment parameters, and initial conditions of the lift-off.

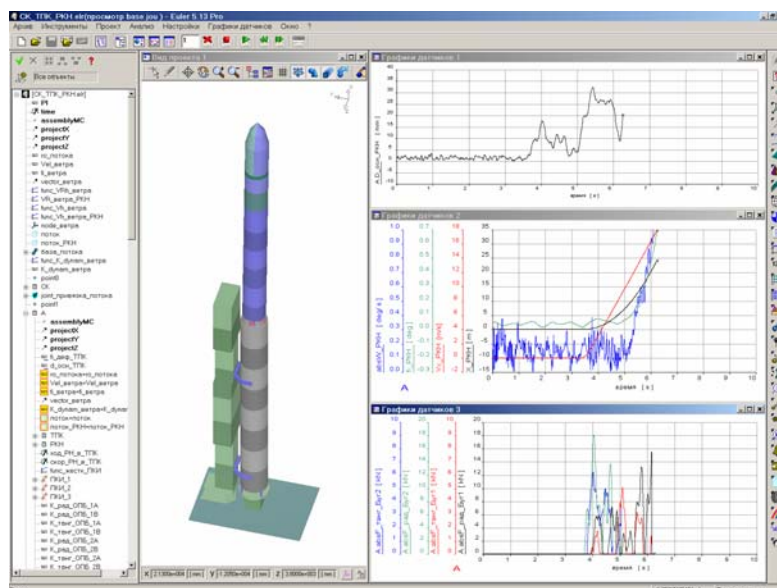


Fig. 1. Simulation of Rocketot Carrier-Rocket Lift-off

Fig. 2 shows model of UAZ-3151 off-road vehicle. The model was used for handling and durability analysis. Fig. 3 shows simulated and measured at road tests time histories of the vehicle motion parameters during lane change. Each plot also contains average discrepancies between measured and simulated data. Fig. 4 demonstrates shots of the simulated motion of the vehicle during lane change.

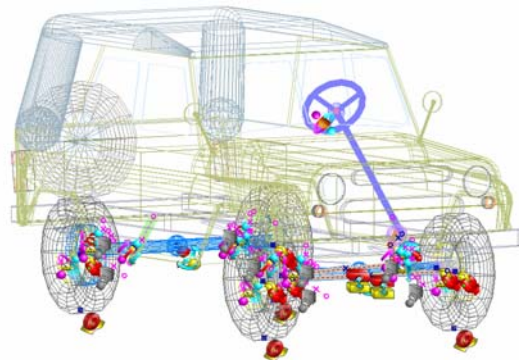
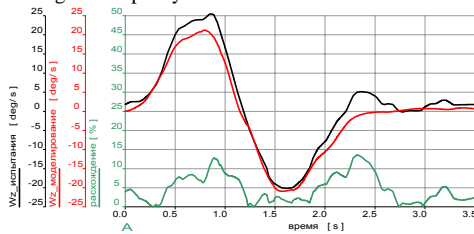
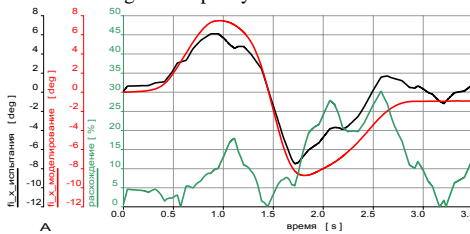


Fig. 2. UAZ-3151 Model

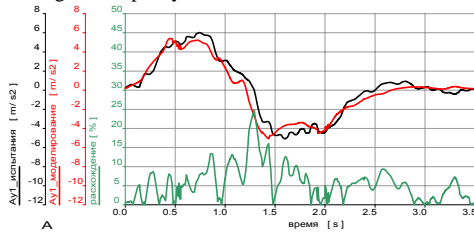
Vehicle Angular Velocity.  
Average discrepancy 5.6 %.



Roll angle.  
Average discrepancy 11.0 %.



Lateral Acceleration above Front Axle.  
Average discrepancy 6.8 %.



Lateral Acceleration above Rear Axle.  
Average discrepancy 6.4 %.

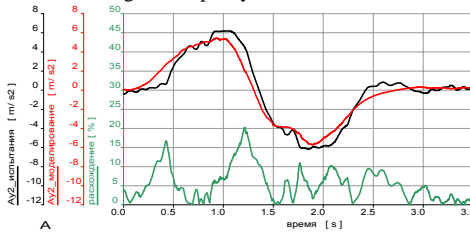


Fig. 3. Comparison of Measured and Simulated Data for UAZ-3151, Lane Change ( $S_p=20$  m) Performed at Velocity 65 km/h.

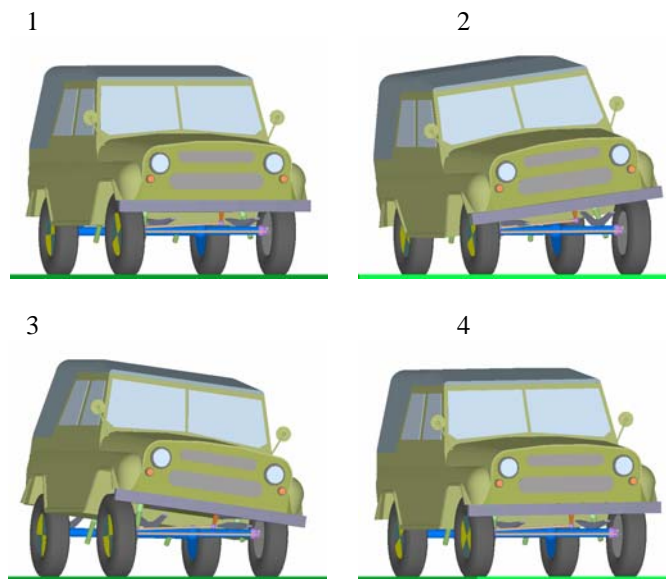


Fig. 4. Shots of the simulated motion of the vehicle

Fig. 5 is a screenshot of EULER environment during analysis of the bell tongue suspension loads. The research was performed for Tsar Bell of Saint Trinity - St. Sergius Lavra.

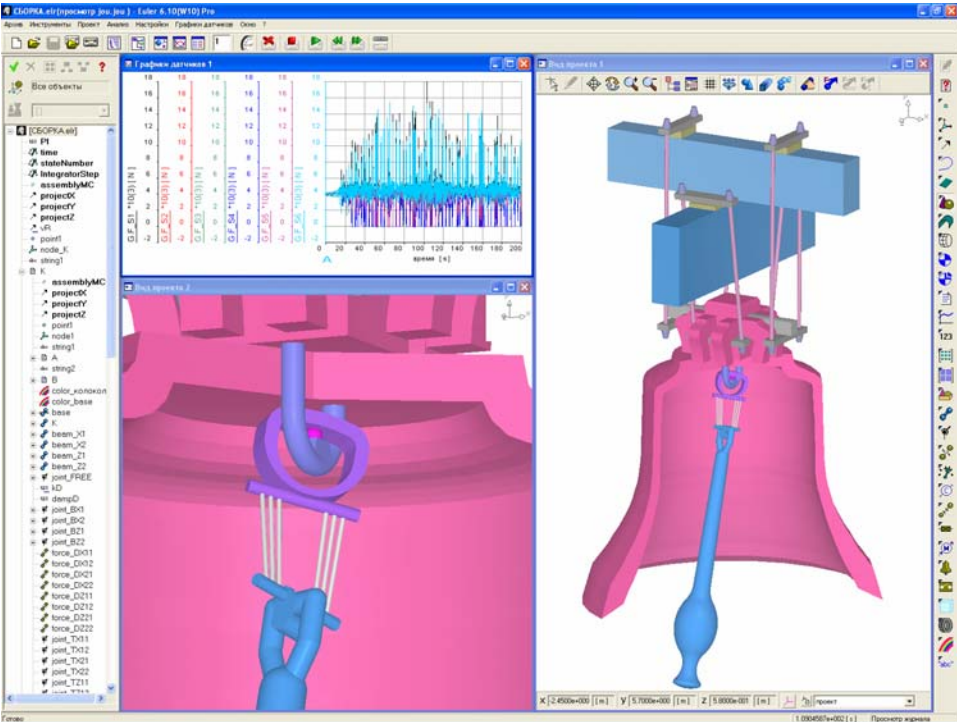


Fig. 2 Analysis of Tsar Bell Tongue Suspension Loads for Saint Trinity - St. Sergius Lavra

EULER is successfully used at designing, improvement, testing and operational development of products, at the analysis of emergency situations, at scientific and applied research, and also in education.

**References**

[1] <http://www.euler.ru>

# Classification of Various Forms of Dynamic Multibody Equations

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**Keywords:** multibody system, equation of motion, quasi-velocities, generalized coordinates, reaction forces, Lagrange multipliers, matrix structure, simulation efficiency.

In this paper the classification is based on the new compact forms of multibody system equations.

Our method is based on two principal points. The first point is: kinematical structure matrix is introduced; it reflects both topological structure of multibody system, and position of the bodies relative to one another. The second point is: a unified geometric approach to modeling of constrained mechanical system is used. The constraints are used to define two canonical subspaces, the constraint force subspace in the cotangent space and the three velocity subspace in the tangent space of the constraint manifold. It is supposed that these spaces can be represented parametrically, by Lagrange multipliers and generalized coordinates of relative motion. These two points make it possible to derive multibody dynamic equations in compact matrix notation.

The relations among two subspaces, kinematic equations and Newton-Euler equations for single rigid bodies form the redundant set of equations. This set of equations consists of differential and algebraic equations in redundant set of coordinates: quasi-velocities of bodies, generalized coordinates of relative motion, reaction forces of constraints or Lagrange multipliers. The redundancy of coordinates and equations originates a variety of multibody formulations. In the report the attempt is made to get a collection of various forms of dynamic equations with the usage of different coordinate's sets. All forms of equations are turned out in a common notation to facilitate their presentation and comparison.

The generated equations are considered from the point of simulation efficiency: the quantity of operations, time costs of computational modeling. Various forms of dynamic equations are studied according to system matrix structure. Matrix structure plots are the basis of the sparsity pattern of the system matrices analysis. Matrixes of special types are more preferable (symmetric, symmetric positive definite, block-tridiagonal, block-band).

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# Modeling and control of vibration-driven mobile systems with internal masses

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**Keywords:** Bodies with movable internal masses, vibration-driven systems, dynamics, control.

## 1. Introduction

The subject matter of the paper is the modeling and control of multibody mechanical systems that can move in a resistive medium without specific propelling devices, such as wheels, legs, fins, caterpillars, or screws. Such systems consist of a body with movable masses inside. The internal masses interact with the body by means of the forces generated and controlled by drives. When the control force is applied to an internal mass, the reaction force is applied to the body and changes its velocity, which affects the resistance force exerted on the body by the environment. Thus, the control of motion of the internal masses provides the control of the external force acting on the body and enables one to control the motion of the entire system. This principle of motion can be used in mobile robots, especially in mini- and microrobots designed for the motion inside small-diameter tubes and in vulnerable media. The body of such robots can be made hermetic and smooth, without protruding components. This enables such robots to be used for medical purposes, for example, for the motion inside blood vessels or the digestive system to deliver a drug or a diagnostic sensor to an affected organ. The principle of motion under consideration can provide high positioning accuracy for mobile devices based on it. Usually, regular progressive motions of such system are generated and sustained by periodic vibratory motions of the internal masses. For that reason, it is appropriate to call such mobile systems the vibration-driven systems. In the present paper, we consider vibration-driven systems the body of which moves in a resistive medium translationally along a horizontal line, while the internal masses move in a vertical plane passing through this line. We construct periodic motions of the internal masses so as to provide a velocity-periodic progressive motion of the body. The motion modes of the internal masses are optimized to maximize the average speed of the body. The cases where the body moves in a viscous medium or along a rough plane (with Coulomb's frictions acting between the body and the plane) are considered.

## 2. Vibration-driven system with one internal mass

The system consists of the body and one internal mass. Both the body and the mass move horizontally along the same line. The body is surrounded by a viscous medium or moves along a dry rigid rough plane. Velocity-periodic progressive motions of the body sustained by periodic motions of the internal mass are studied for various nonlinear laws of resistance of the environment to the motion of the body. In the case of the motion of the body along a dry surface, the friction force is assumed to obey Coulomb's law. Two types of periodic motions of the internal mass are considered, two-phase and three-phase motions. For the two-phase motion, the relative velocity of the internal mass is a piecewise function of time with two intervals of constancy for the period. For the three-phase motion, the relative acceleration of the internal mass is a piecewise constant function of time with three intervals of constancy for the period. The maximum magnitudes of the relative velocity or acceleration of the internal mass, as well as its maximum displacement relative to the body, are subjected to constraints. The values of the relative velocity (acceleration) of the internal mass on the intervals of constancy, the durations of these intervals, and the period of motion are the parameters to be determined. These parameters are chosen so as to maximize the average velocity of the body.

## 3. Three-body system moving along a rough plane

The system consists of the body and two internal masses. One of the internal masses moves horizontally along the line of motion of the body, while the other mass moves vertically. There is Coulomb's friction force acting between the body and the plane. The friction force can be controlled by only one mass that moves horizontally. However, the horizontal motion of the internal mass does not ensure the control of the normal pressure force that also influences the magnitude of Coulomb's friction force. The introduction of the mass allowed to move vertically provides such a possibility and, hence, increases the control capabilities. For this system, an optimal control problem is solved. The role of the control variables is played by the accelerations of

the internal masses relative to the body. The absolute values of these accelerations are constrained due to restricted power of the drives. It is assumed that contact between the body and the plane is not violated, which implies an additional upper constraint on the magnitude of the downward acceleration of the mass moving vertically. Periodic motions of the internal masses are constructed that satisfy the constraints, provide a velocity-periodic motion for the body, and maximize the displacement of the body in the desired direction for a fixed period. It is proven that the period consists of two intervals. The body moves forward during the first interval and remains fixed during the second interval. In the optimal motion, the body never moves backward. On the first interval, each control is of bang-bang type with one switching instant at most. On the second interval both controls are constant. The optimal control problem is reduced to the constrained maximization of a quadratic function. The parameters to be found are the duration of the interval of the forward motion of the body and the switching instants of the controls. This statement of the control problem does not impose constraints on the maximum relative displacements of the internal masses. Such constraints can be taken into account by varying the period of the motion. Detailed solutions are obtained for two limiting cases, where the vertical motion of the mass is prohibited and where the relative acceleration of this motion is constrained only by the condition that the body does not lose contact with the plane. In the second case, the displacement of the body for the period is more than 4 times the displacement provided by the only internal mass moving horizontally. This confirms the reasonability of introducing vertically moving internal masses in the design of vibration driven-systems for motion along rigid dry rough surfaces.

# A formal procedure and invariants of a transition from conventional finite elements to the absolute nodal coordinate formulation

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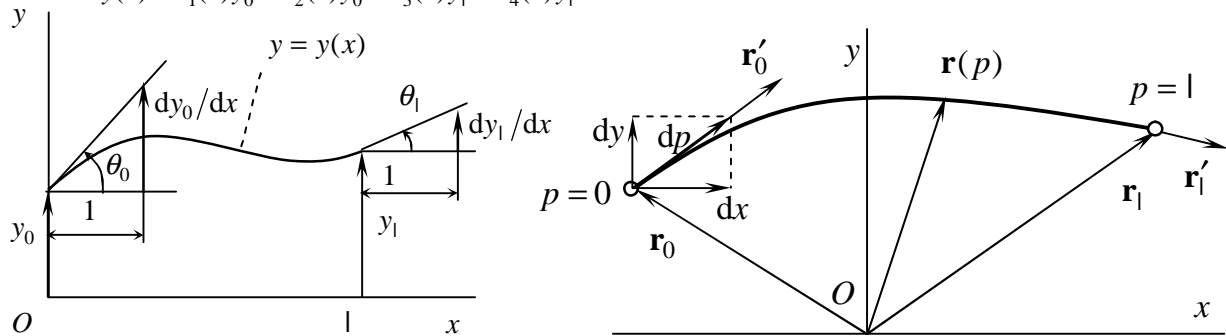
## Introduction

In this paper, the finite elements based on the absolute nodal coordinate formulation (ANCF) are studied. The formulation has been developed by various authors for dynamical simulation of large-displacement and large-rotation problems in the flexible multibody dynamics. An attempt is made to track the general geometrical properties of elements based on the formulation back to their prototypes in conventional finite-element method (FEM), which deals with small-displacement problems. In this study, it is shown that each known ANCF element can be derived from a convenient FEM using a universal transform. Moreover, some important static and dynamic properties of the elements in small-displacement problems are automatically preserved. In the past, the authors of each newly proposed ANCF element have been paying unnecessary efforts to show the consistency of the above mentioned properties.

## The formal procedure

The idea of the general rule is explained by giving a simple example of the conventional planar beam based on Euler-Bernoulli theory, Fig. 1, left-hand side. It has two degrees of freedom per each node: transverse displacements  $y_0$  and  $y_l$  as well as nodal slopes represented by derivatives  $y'_0 = dy_0/dx = \tan \theta_0$  and  $y'_l = dy_l/dx = \tan \theta_l$ ; the displacements and slopes are assumed to be small. Displacement of an arbitrary point  $x$  is

$$y(x) = S_1(x)y_0 + S_2(x)y'_0 + S_3(x)y_l + S_4(x)y'_l.$$



**Fig. 1. Conventional beam element with small displacements and large-displacement ANCF element [15]**

In case of large displacements, the arc coordinate  $p$  instead of spatial coordinate  $x$  is introduced,  $x \rightarrow p$ . Accordingly, two independent fields for coordinates  $x$  and  $y$  are defined using the same shape functions:  $y_0 \rightarrow \{x_0, y_0\} = \mathbf{r}_0$ ,  $y_l \rightarrow \{x_l, y_l\} = \mathbf{r}_l$ . The slopes that have been tangent functions become directing cosines and, consequently, they form the tangent vectors:  $\frac{dy_0}{dx} \rightarrow \left\{ \frac{dx_0}{dp}, \frac{dy_0}{dp} \right\} = \frac{d\mathbf{r}_0}{dp} = \mathbf{r}'_0$ ,  $\frac{dy_l}{dx} \rightarrow \left\{ \frac{dx_l}{dp}, \frac{dy_l}{dp} \right\} = \frac{d\mathbf{r}_l}{dp} = \mathbf{r}'_l$ . After the transformation, the position vector of an arbitrary point of the beam can be expressed as follows:

$$\mathbf{r}(p) = S_1(p)\mathbf{r}_0 + S_2(p)\mathbf{r}'_0 + S_3(p)\mathbf{r}_l + S_4(p)\mathbf{r}'_l.$$

This procedure can be considered as the immersion of 1-manifold into 2D space, the number of degrees of freedom  $N$  of the source element is multiplied by  $m=2$ , such that the new number of d.o.f. is  $\mathcal{N} = N \times m$ . The immersion into 3D space will multiply it by  $m=3$ . The full list of known implementations of structural ANCF elements is given in Table 1, right-hand column. The corresponding conventional elements are presented in the left-hand column. This list can be extended by adding any existing structural element to the left-hand side.

It can be shown that some properties are preserved after such transformation: 1) same strains at any rigid-body rotation; 2) same small displacements; 3) same natural frequencies at initial configuration.

**Table 1. Mapping of the structural finite elements of beams and plates from conventional FEM to ANCF**

Elem. type	Conventional FEM elements		$\times m$	Elements adopted for ANCF	
	Year, Author(s)	$N$		$= \mathcal{N}$	Year, Author(s)
–	???? Standard element [?]	4	2	8	1996 Shabana [15]
			3	12	2006 Dmitrochenko, Yoo & al. [4]
–	1974 Narayanaswami & Adelman [12]	4	2	8	2007 Mikkola, Dmitrochenko & al. [9]
□	1966 Bogner, Fox & al. [1]	16	3	48	2003 Dmitrochenko & Pogorelov [5]
□	1963 Melosh [8]	12	3	36	2005 Dufva & Shabana [6]
△	1988 Specht [17]	9	3	27	2007 Dmitrochenko & Mikkola [3]
△	1971 Morley [11]	6	3	18	2007 Dmitrochenko & Mikkola [3]
◇	1968 Clough & Felippa [2]	12	3	36	2007 Schwab, Gerstmayr & al. [14]

## Conclusions

This paper discovers relations between conventional and ANCF structural elements that use Euler–Bernoulli or Kirchhoff theory for beams and plates.

It is expected that the same relations hold for so-called fully parameterized ANCF elements that have been recently proposed: [7], [10], [13], [16]; they employ Timoshenko and Reissner–Mindlin theories. However, during this research, the authors have failed to find conventional elements that can correspond to the latter ones.

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# Generalities of Passive Vibration Dampers Isolating Vibrations

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**Keywords:** passive uniaxial vibration isolator, passive shock-absorbers, differential operator, integral Laplace transform.

## 1. Introduction

The vibrations of mechanical systems analysis is widely used in modern mechanical engineering at designing vibration shock absorbers, vibration isolators, aviation technics, a rolling stock of the underground, trains, machine tools. One of existing approach of the analysis is based on use of a method of chain dynamic systems, or full mechanical resistance - an impedance [1] resulting to simplification of mathematical model of a problem due to acceptance of special assumptions and limitations on dynamic and kinematic parameters of real object, attracting infringement of dynamic processes logic and laws newtonian mechanics [2]. Other method of the oscillatory systems researches, considering most authentically physical laws of investigated mechanisms, is the method of amplitude-frequency characteristics [3]. Its essence consists in exact mathematical modelling dynamic movements of multibody mechanical object, the decision of the linear differential equations system by use integral Laplace transform. It allows to analyse mechanical system with use of transfer functions, frequency characteristics and to receive required decisions under Riemann-Mellin formulas, what frequently to carry out it happens rather difficultly.

In the present work the method of research linear and linearized oscillatory systems with a plenty of the required variables is submitted, allowing to divide the associated differential equations of the second order into the independent linear non-uniform differential equations, to receive exact decisions, a characteristic polynomial, to find amplitude -frequency characteristics, transfer functions, to determine stability of system and “quality”, “stock of stability”.

## 2. Theoretical results

As oscillatory system we shall consider passive uniaxial vibration isolators mobile means. Physical properties of substances and materials of passive shock-absorbers are stable, do not change eventually as against active, having electro-rheological or magneto-rheological liquids in the designs. It will consist of metal springs, salient blocks, rubber-metal shock absorbers, hydrosupport. Preliminary designing of such mechanisms is connected to use of the elementary elastic and damping elements linearly dependent on movings and speeds. The general vibration isolation model can be submitted by the dynamic scheme on Fig.1 which equations of movements are expressed by four associated non-uniform differential equations with not divided variables.

$$\ddot{x}_1 = -b_{11} \dot{x}_1 + b_{12} \dot{x}_2 + b_{13} \dot{x}_3 - c_{11}x_1 + c_{12}x_2 + c_{13}x_3 + \Phi_1, \quad (1)$$

$$\ddot{x}_2 = b_{21} \dot{x}_1 - b_{22} \dot{x}_2 + b_{23} \dot{x}_3 + c_{21}x_1 - c_{22}x_2 + c_{23}x_3 + \Phi_2, \quad (2)$$

$$\ddot{x}_3 = b_{31} \dot{x}_1 + b_{32} \dot{x}_2 - b_{33} \dot{x}_3 + c_{31}x_1 + c_{32}x_2 - c_{33}x_3 + \Phi_3, \quad (3)$$

where  $\Phi_1 = f_1$ ,  $\Phi_2 = f_2$ ,  $\Phi_3 = f_3$ .

The equations of such type frequently meet in mechanics. They describe oscillatory movements of a ship and the ship gyroscope, the friction and constrained oscillatory systems, absolute movements of a horizontal pendulum. It is not obviously possible a general view to integrate system of the differential equations. There are some decisions for specially picked up right parts and conditions, imposed on factors  $b_{ij}$ ,  $c_{ij}$  ( $i = 1, 2$ ;  $j = 1, 2$ ) [4]. Modern computer means allow to receive numerical results for small time intervals and concrete parameters of a problem from which character of influence of factors on the general oscillatory process is not seen. The discrepancy of system of the differential equations (1) - (3) results in accumulation of an error and the discontinuance of the account for the limited amount of steps. A problem of system (1) - (3) is connection of variable, their first and second derivatives. Let's divide derivatives, using operation of repeated differentiation which does not contradict the theorem of existence and uniqueness Peano [4]. For this purpose we shall allocate from system of the equations (1) - (3) following differential operators of the second and first orders.

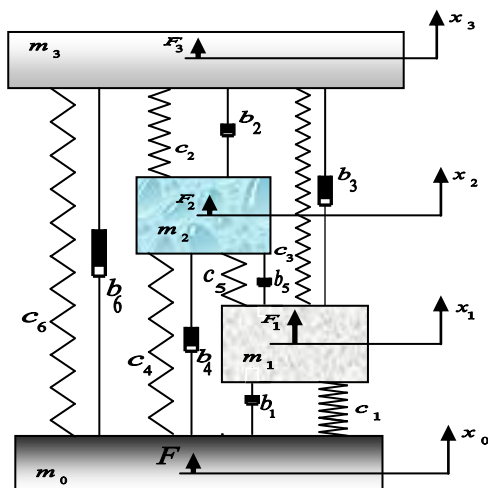


Fig. 1: The general vibration isolation scheme

$$L_1 = \frac{d^2}{dt^2} + b_{11} \frac{d}{dt} + c_{11}, \quad L_2 = \frac{d^2}{dt^2} + b_{22} \frac{d}{dt} + c_{22}, \quad L_3 = \frac{d^2}{dt^2} + b_{33} \frac{d}{dt} + c_{33}, \quad (4)$$

$$d_{12} = b_{12} \frac{d}{dt} + c_{12}, \quad d_{13} = b_{13} \frac{d}{dt} + c_{13}, \quad d_{21} = b_{21} \frac{d}{dt} + c_{21}, \quad (5)$$

$$d_{23} = b_{23} \frac{d}{dt} + c_{23}, \quad d_{31} = b_{31} \frac{d}{dt} + c_{31}, \quad d_{32} = b_{32} \frac{d}{dt} + c_{32}.$$

The dynamic system of vibration isolation (1) - (3) can be submitted thus:

$$L_1(x_1) = d_{12}(x_2) + d_{13}(x_3) + \Phi_1, \quad L_2(x_2) = d_{21}(x_1) + d_{23}(x_3) + \Phi_2, \quad L_3(x_3) = d_{31}(x_1) + d_{32}(x_2) + \Phi_3. \quad (6)$$

Consistently applying operators  $L_i$  ( $i = \overline{1,3}$ ) to the differential equations (6) concerning variables, we receive new system of three incoherent differential equations of the sixth order.

$$L_1 L_2 L_3(x_i) - L_1 d_{23} d_{32}(x_i) - L_2 d_{31} d_{13}(x_i) - L_3 d_{21} d_{12}(x_i) - d_{12} d_{23} d_{31}(x_i) - d_{13} d_{32} d_{21}(x_i) = L_j L_k(\Phi_i) - d_{jk} d_{kj}(\Phi_i) + L_j d_{ik}(\Phi_k) + d_{ij} d_{jk}(\Phi_k) + L_k d_{ij}(\Phi_j) + d_{ik} d_{kj}(\Phi_j), \quad i = \overline{1,3}; \quad j = \overline{1,3}; \quad k = \overline{1,3}, \quad i \neq j \neq k, \quad (7)$$

if  $i = 1$ , then  $j = 2$ ,  $k = 3$ ; if  $i = 2$ , then  $j = 3$ ,  $k = 1$ ; if  $i = 3$ , then  $j = 1$ ,  $k = 2$ .

The system of the equations (7) allows to determine obviously constant factors of the linear inhomogeneous equations of the sixth order which integrating comes easily. Decisions of system (1) - (3) are searched on the basis of integral Laplace transform application.

### 3. Conclusions

The developed technique of multibody dynamic mechanisms research with the help of consecutive application of differential operators allows to lower toilful mathematical calculations, to estimate their "quality" and stability of movements on known factors of general mathematical model (1) - (4). The offered approach will be coordinated to methods of integral Laplace transform, variations stationary amplitudes, chain dynamic systems and an impedance. Its advantage is the opportunity of a finding of required movings and speeds of weights as decisions of the linear inhomogeneous differential equations independent from each other. It gives an additional condition for check of numerical calculations as allows to represent evidently oscillatory movements of weights in a phase plane, to characterize a kind of phase trajectories, type of special points and to determine position of mechanical oscillatory system balance.

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# Breaking of Particle Connections in Multibody Systems

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**Keywords:** Particle, force laws, cohesive material, machine tools, cutting.

The discrete element method (DEM) has been widely used in the past to model granular materials like powder and sand. In recent years this approach was adopted by engineers in geomechanics and civil engineering to model geomaterials like concrete or marl. However, most modelled geomaterials exhibit very brittle behaviour when loaded. Here, it is investigated if the so far used approach can be advanced to model materials that exhibit ductile behaviour to some extent as well. Different models based on the DEM for the simulation of cohesive materials are presented. Starting from a basic two-dimensional non-cohesive model with circular particles the model is enhanced by including massless beam elements to introduce cohesive forces between adjacent particles. The particle bonds are represented by force laws and can sustain only a specified stress until failure. By implementing different force laws and failure criterions it is investigated if the composite made up of bonded elements displays the desired failure modes.

Various different phenomena like, e.g. stress wave propagation, elastic-plastic deformation or breaking can be observed when loading mechanical structures. Traditionally used methods to analyze these scenarios like FEM or BEM solve partial differential equations of continuum mechanics. If fracture and fragmentation are to be incorporated in the scenario it is difficult to deal with the resulting discontinuities while using these classical methods. A different approach to model damage and failure is based on the DEM. Using this approach, the material is considered as being fully discontinuous and made up by assembling and bonding adjacent discrete elements. Different types of force laws can be used depending on the considered problem to be modeled as well as the type of particles used.

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# Dynamic modeling and analysis of the human knee joint

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**Keywords:** Knee joint, ligaments, dynamics, contact analysis, multi-body dynamics.

## 1. Introduction

The biomechanics studies of the human knee joint have received a great deal of attention in the past, and still remains an active field of research and development [1-3]. The dynamic modeling of the intact human knee joint is presented and analyzed throughout this work. The proposed model is developed under the framework of the multibody systems methodologies. The femur and tibia bones are considered as rigid bodies, being their articular cartilages modeled as deformable elements. The shapes of the femur and tibia are obtained from magnetic resonance image technique, defined in the sagittal plane. After digitalizing the images produced, the outlines of the profiles are discretized and described in polar coordinates. Then, the cubic interpolation splines approach is used. The adoption of this solution is of paramount importance in order to ensure continuity in the first and second function derivatives. Thus, based on the kinematic configuration it is possible to evaluate if the femur and tibia are in contact with each other. When that contact occurs, a continuous constitutive law is applied in order to compute the contact forces produced by the contact. Then, these resulting forces are introduced into the system equations of motion as external generalized forces. These contact forces are dependent on the relative deformation between femur and tibia cartilages, as well as on the contacting surface properties, such as Young's modulus and Poisson's ratio. In addition, in the present study, the four basic ligaments that exist in the knee articulation are modeled as non-linear elastic springs. The patello-femoral and menisci are not included in the present knee model.

The main features that characterize and distinguish the model proposed here are: (i) the model is dynamic, since it relates the body forces with motion produced, being the dynamic models more appropriate for studying human daily activities than are static models; (ii) this model explicitly relates the knee mechanical properties and contact forces produced, what is not the case in the most of the models available in the literature; (iii) the model is simple, generic and easy to implement in other types of biomechanical systems, such as those that consider whole-body human system. Finally, a simple computational simulation is used to demonstrate the efficiency of the methodology proposed, as well as to discuss the assumptions and procedures adopted.

## 2. Knee joint description

The knee joint is one of the biggest and more complex synovial joints that exist in the human body, which main functions are: (i) to permit the movement during the locomotion and (ii) to allow the static stability. The mobility associated with the knee joint is indispensable to human locomotion and to help the correct foot orientation and positioning in order to overcome the possible ground irregularities. In the knee articulation there are three types of motion, namely, flexion, rotation and sliding of the patella. The knee joint includes three functional compartments, medial, lateral and patello-femoral, which makes the knee quite susceptible to injuries, chronic disease, such as displacement, arthritis, rupture's ligaments and menisci separation. In fact, the greatest number of human ligament injuries occurs to ligaments of the knee. A joint capsule surrounds the knee joint with ligaments strapping the inside and outside of the joint (collateral ligaments) as well as crossing within the joint (cruciate ligaments). The collateral ligaments run along the sides of the knee and limit the sideways motion of the knee. The anterior cruciate ligament connects the tibia to the femur at the centre of the knee and functions to limit rotation and forward motion of the tibia. The posterior cruciate ligament located just behind the anterior cruciate ligament limits the backward motion of the tibia. All of these ligaments provide stability and strength to the knee joint.

In short, the knee as living, self-maintaining and biologic transmission system has the purpose to support and transmit biomechanical loads between femur, tibia and fibula. In this analogy, the ligaments represent non-rigid bodies adaptable within the biologic transmission system. Finally, the articular cartilages act as fixed bearing surfaces, while menisci act as mobile bearings. The muscles function as living cellular engines that in concentric contraction provides active forces across the joint, and in eccentric contraction act as brakes and damping systems, absorbing and dissipating loads.



### 3. Modeling knee joint

The purpose of this section is to present a simple model for the intact human knee joint developed under the framework of multibody systems formulation. Figure 1 shows two bodies  $i$  and  $j$  that represent the tibia and femur, respectively. Body-fixed coordinate systems  $\xi\eta$  are attached to each body, while  $XY$  coordinate frame represents the global coordinate system. The origin of the femur coordinate system is located at the intercondylar notch, while the origin of the tibia coordinate system is located at the centre of mass of the tibia, being the  $\xi_i$ -axis directed proximally and  $\eta_i$ -axis directed posteriorly. These origin points are represented by points  $O_i$  and  $O_j$ . The angles of rotation of the local coordinate systems of bodies  $i$  and  $j$ , relative to the global system, are denoted by  $\phi_i$  and  $\phi_j$ , respectively.

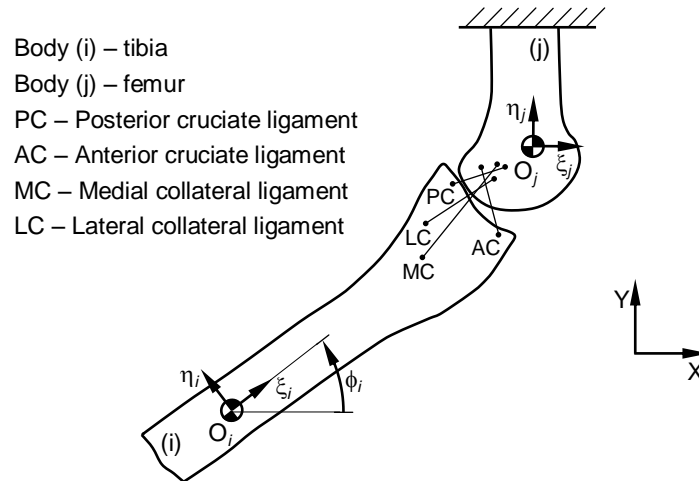


Figure 1. Representation of the knee joint including the femur and tibia elements and the four primary ligaments.

In the present work, the femur and tibia elements are modeled as two contacting bodies, being their dynamics controlled by contact-impact forces. The equations of motion that govern the dynamic response of this multibody system incorporate these types of forces. Furthermore, the knee joint elements are considered to be rigid and describe a general planar motion in the sagittal plane. The femur is considered to be fixed, while the tibia rolls and slides in relation to the femur profile. The femur and tibia are linked by four nonlinear elastic springs in order to represent the knee joint ligaments, as it is illustrated in Fig. 1.

With the intent to develop a mathematical model for the human knee joint that allows to perform dynamic analysis, it is first necessary to define with accuracy the shapes of the femur and tibia profiles. In this work, a magnetic resonance image (MRI) of the knee articulation in the sagittal plane is used to obtain those profiles. Then, based on the MRI image, two sets of points are considered on the articular cartilages of the femur and tibia bones. In order to describe these outlines in closed-form expressions, cubic interpolation splines functions are used, which consist of polynomial pieces on subintervals joined together according to certain smoothness conditions. For this purpose, the degree selected for the polynomial functions is 3, being the resulting splines named cubic splines. The reason for that is due to the fact that the cubic polynomial functions are joined together in such a way that they have continuous first and second derivatives everywhere.

The proposed model for the knee joint includes two rigid bodies, namely the femur and the tibia, which accounts for the deformability of their articular cartilages. The mathematical formulation is based on the profiles of femur and tibia, which defines the geometric condition for detection of contact between the two bodies. A continuous constitutive contact force law is used to evaluate the contact forces produced during the contact between femur and tibia surfaces.

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# Computer Simulation of the Six Legged Robot's 3D-Dynamics into the Media with Obstacles

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**Keywords:** dynamics, simulation, climbing, robot, friction.

## 1. Introduction

Development of methods of climbing robot's motion along the complex surfaces with the obstacles of sizes exceeding the size of the robot is an actual contemporary problem. Universal algorithms to deal with any possible surfaces are not yet developed. In the presented work we take some typical obstacles that robot can meet in some industrial media and develop the algorithms to overcome the sequence of them. By using the virtual, software environment we can find out the principal parameters required to build the real, "iron" robot, and optimize appropriate control algorithms.

## 2. Main body

Results presented in the report continue the research contained in [1]–[3]. Algorithms for overcoming a sequence of high obstacles by an insectomorphic (six legged) climbing robot are developed. The sequence consists of a vertical cylindrical column and a lofty horizontal shelf with a vertical wall, two identical lofty horizontal shelves connected by a narrow horizontal beam; vertical right corner with a horizontal upper bench surface; a ladder leaned against a vertical wall of the shelf. All obstacles have a vertical plane of symmetry.

The assumptions taken to create a computer model are as follows.

All bodies mentioned in this abstract (parts of the obstacles or the robot) are rigid. The robot consists of parallelepiped-shaped main body and six identical legs, attached uniformly and symmetrically to it. Each leg consists of two links and has 3 degrees of freedom. Two joint angles define the position of upper link related to the body and another one is the angle between links, i.e. the angle in the knee. Thus, the robot has 24 degrees of freedom. It can contact with a supporting surface only by feet of the lower links. The robot doesn't have vacuum suckers and can use only friction forces at supporting points.

The reaction of the surface is calculated as an elastic-viscous force with friction cone restriction.

The robot has the electromechanical drives in joints aimed to realize the programmed values generated by the control algorithm. Drives are modeled as the linear PD-regulators [4], and thus the robot's motion is stable to the small enough variations of feet and body positions related to the programmed values. Control algorithm can access at any instant full and precise information about the geometry of the obstacles, robot's position relatively to them, current joint angles. Standard gravity force affects the system.

Control algorithms are built on the assumption that the height of the obstacles doesn't permit the robot to overcome them without climbing on the outer surface of the considered obstacle or using another one to climb over. In particular, the height of the shelf's plane wall doesn't permit for robot to climb on it without using the column, the corner or the ladder.

Algorithm for the case of climbing to the shelf using the column has several branches depending on difference of the column's and the shelf's heights. All possible differences that robot can overcome due its size are considered. For this case, and below, if not stated evidently, the friction coefficient was taken as less or equal to 1.

During the motion along the horizontal beam [2]–[3] the width of the beam is assumed much less than the width of the robot's body. It doesn't allow creating big enough cross static stability margin. In this case the quadruped diagonal gait was used. It allows for robot to move with the help of front and rear legs without intersection by using the narrow footstep rut under the body. When the diagonal pair of legs is supporting, the body moves like a physical pendulum fixed on the axis that passes through the supporting points and is located at

the upper unstable equilibrium position. To stabilize this position we use the pair of middle legs that performs a coordinated rotation in the plane perpendicular to the body axis.

For the climbing along the right vertical corner to its upper bench surface it's impossible to accomplish the motion with friction coefficient 1, so it was raised to 1.1, and an appropriate control algorithm was designed. Middle legs are transferred two times more often than front and rear ones, and each time they are placed near the pair that will be transferred next. Thus, the big enough lever between supporting points is achieved to prevent the fall from the surface.

For the case of climbing along the ladder robot can use supporting points only on separated crossbeams. This differ the motion on the ladder from other obstacles because there is no possibility for adaptation of feet positions along the motion direction. At some stages of motion middle legs have to go back to support on previous crossbeams due to save the static stability (as in motion up the vertical corner). Sometimes rear legs have to be transferred one by one.

For all cases control algorithms include adaptation of motion to the obstacles parameters. For example, to deal with the possible different heights of the vertical angle, robot strikes some standard attitude when it approaches to the area near the upper surface. Then it proceeds with the maneuver of turning to the horizontal position with known parameters.

The 3D computer simulation was fulfilled using program package Universal Mechanism [5] to elaborate control algorithms for its effectiveness and robustness. Some results of the computer simulation of robot overcoming indicated obstacles will be given in the presentation.

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# THE METHODS OF THE TRANSFORM OF MULTIBODY SYSTEMS WITH REDUNDANT CONSTRAINTS

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The mechanical systems of arbitrary structures with closed loops and redundant constraints are investigated. A method of transformation of redundant constraints to exclude static indefinite is proposed, which provides a predefined accuracy of numeric solution. The method is based on introduction of some kinematic subchain within the full kinematics scheme of machine. Redundant constraints sets are found by special assemble algorithms. The proposed theoretical method was implemented in the multibody dynamics simulation program FRUND. The paper presents the results of computer simulation of some examples such as a spatial mechanical system and CLAWAR (walking machine).

The most of the publications devoted to the problem of redundant constrains raised from the domain of the robotics control motion [1]. In the problems of mechanical system dynamics the redundant constraints are analyzed to be detected and excluded [2]. The excluding of the redundant constraints may lead to a distortion of the dynamic parameters of systems, for example the reactions at links. So it is important to develop new methods of the dynamic simulation for the systems with redundant constrains.

Solutions of the described problems can be found numerically from the equations, which can be obtained in the form of Lagrange equations of 1<sup>st</sup> type:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}} - \mathbf{D}^T \mathbf{p} = \mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}, t) \\ \mathbf{D}\mathbf{x} = \mathbf{h}(\dot{\mathbf{x}}, \mathbf{x}) \end{cases} \quad (1)$$

Here  $\mathbf{x}$  is the state vector of dimension  $n$ ,  $\mathbf{M}$  is the inertia matrix,  $\mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}, t)$  is the vector of external forces,  $\mathbf{D}$  is the  $k \times n$  matrix of the variable coefficients of constraints equations,  $k$  is the number of constraints equations,  $\mathbf{h}(\dot{\mathbf{x}}, \mathbf{x})$  is the right hand side terms of constraints equation,  $\mathbf{p}$  is the vector of Lagrange multipliers.

In the case  $rank(\mathbf{D}) = k_1 < k$  we have redundant constraints, and numeric integration of (1) is impossible. The propose of the developed method is to modify of the matrices  $\mathbf{M}$  and  $\mathbf{D}$  as

$$\mathbf{M}_e = \begin{pmatrix} \mathbf{M} \\ \mathbf{M}_s \end{pmatrix}, \quad \mathbf{D}_e = \begin{pmatrix} \mathbf{D}_1 \\ \mathbf{D}_s \end{pmatrix} \quad (2)$$

where  $\mathbf{M}_s$  is the inertia matrix of the subchain bodies,  $\mathbf{D}_1$  is the matrix  $\mathbf{D}$  without  $(k - k_1) * 2$  constraints equations due to redundant links and some deleted constraints,  $\mathbf{D}_s$  is the matrix of the variable coefficient in constraints equations corresponding to links of the subchain bodies. The dimension of  $\mathbf{D}_e$  matrix is  $k_2 \times n$ , and  $rank(\mathbf{D}_e) = k_2$ . The estimate of the accuracy for the Lagrange multipliers  $\mathbf{p}_e$  of the modified system may be written in the form  $|\mathbf{p} - \mathbf{p}_e| \sim |\mathbf{M}| / |\mathbf{M}_s|$ . The matrix norm here is the maximal absolute value of matrix elements.

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# Wear of wheel and rail in the process of vehicle-track interaction

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**Keywords:** wheel/rail wear, railway vehicle dynamics.

## 1. Introduction

Serious problems of wheel and rail profile design arise particularly when combined freight and passenger traffic exists on the line or this task should be solved on a big railway network scale. The main purpose of such design is the reduction of wheel and rail profile wear and the improvement of operational safety. The technique of solving such problems as well as profile selection policies and real practices by using the multibody approach is described. An example of practical approach to profile evaluation is given. A scientific approach to profiles evolution and optimization is described and some results are presented.

## 2. Main body

There is a wide spectrum of conditions and environment characteristics to impact traffic operation. These are dedicated lines, mixed (combined) passenger and freight traffic, difference in traffic density, terrain conditions, share of curves and tangent track, etc. It is one task when wheel and rail profiles are to be selected for dedicated lines, e.g. heavy haul line, or high speed line. It is much more difficult to solve the problem of profile selection when combined freight and passenger traffic exists on the line on which different type of rolling stock operate or this task should be solved on a network scale. Such a problem exists for the Russian Railways that have about 86,000 km route length and about 124,000 km track length, with several climatic regions, terrain features, and difference in profiles for locomotive, freight, passenger cars and electric trains.

Many studies have been performed on wheel or rail or both profiles evolution and prediction using numerical methods based on dynamic models of vehicle/track interaction, contact mechanics and tribology consideration. These studies enable the evolution of profiles for a particular track and rolling stock and to evaluate the influence of essential factors. Indices for the evaluation of wheel-rail profiles could be local and integral. Local indices are related to the contact area, for instance, to the maximal contact pressure. Integral indices are functionals that could be found during vehicle-track interaction on a representative rail section, for instance, the total wear of the wheel and the rail, or safety indices. The integral indices could be also those that are found for period of wheel and rail life. Wheel/rail profiles are considered as the concurrent if integral indices are within acceptable range. Wheel/rail profiles are considered as the optimal if one of the indices is reach extreme. For instance, the profiles that provide for the minimal wear rate, under limitation on derailment safety criteria, hunting stability, contact pressure. A separate task is a demonstration of the possibility to reach the optimum and the algorithm of optimization. It requires parameterizing the profile description and demonstrating that the optimal profile is within this range.

The task of creating tribodynamic model has been accomplished as a result of joint efforts of Bryansk State Technical University (BSTU) and Institute of Problems in Mechanics of the Russian Academy of Sciences and Russian Railway Research Institute (VNIIZhT). The base for this work was the program complex Universal Mechanism developed by the BSTU. Wheel profile simulation due to wear was performed as multiversion calculation procedure in which external conditions are changing at the end of the procedure. In our case external conditions are wheel profile that is changing due to wheel/rail interaction on selected line and tribological model of wear. Beta-spline is used for smoothing the wear distribution. Particular attention is given to selection of proper size of wear value or vehicle run to achieve appropriate quality of smoothing. An algorithm providing for required smoothing of worn profile was suggested.

The position of contact points, normal and tangential forces acting in these points as well as step-wise wheel profile evolution are obtained from the tribodynamic model. This information is used by the program of the accumulation of the contact fatigue damage.

# Customized Simulators: Software Structure and Case Study

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**Keywords:** software, real-time multibody formalism, sparse linear systems

## 1. Introduction

The simulation software **alaska** has been developed and applied at the Institute of Mechatronics for the simulation of multibody and electromechanical systems. The areas of application are mechanical engineering, vehicle dynamics, and biomechanics. Besides the need for general-purpose software, there is a growing demand for customized simulators (Fig. 1). A simulator is understood as a piece of software for the simulation of a concrete model or a specific model class. Customers ask for stable models where only relevant data can be modified. There is also need for seamless integration of simulators into existing tool chains. Customized solvers can be developed for simulators. These solvers can be faster and more robust than solvers used for general-purpose tools. A concrete solver and a software structure for the development of simulators will be discussed in this talk.

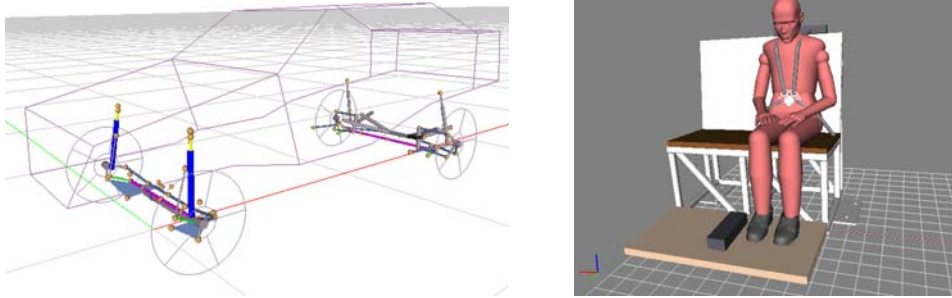


Figure 1: vehicle simulator; crash simulator

## 2. Solver for real-time vehicle models

The objective is the development of a real-time vehicle model with elastokinematic suspension models (Fig. 2, left). The model contains elastic bushings that lead to stiff equations of motion. Implicit integration methods must be used. No additional problems arise in modeling the joints with stiff spring-damper elements. Absolute coordinates for the bodies are used. The equations of motion have a simple structure in this setting. The integration is done using a half-implicit Newmark scheme. The linear system that has to be solved in each step (Fig. 2 right) has a sparse structure. A solver has been developed for the special sparseness structure. This solver is much faster than established solvers in this case. On step of a model with 45 bodies and 80 joints/forces can be integrated in less than 1ms including the time for solving a linear system with 264 unknowns.

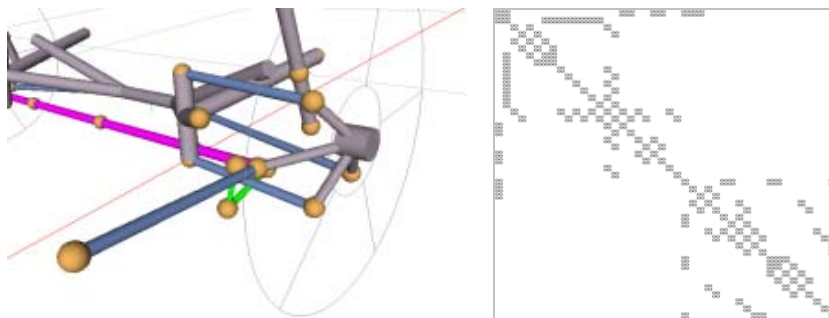


Figure 2: rear suspension; structure of linear system

### 3. Software structure

The basis of the software is a representation of multibody models in C++ together with **alaska/SimulationEngine** – a library for modeling and simulation of multibody systems. The C++ model is not specific to any multibody formalism. New methods can be implemented independently. In the case of automatic code generation only the model is generated, not the equations of motion. Therefore different solvers can be used for one generated model. The model is readable and editable in the sense that new model elements as bodies or forces can be added.

# Iteration Methods of the Solution for Accelerations of the Multibody Systems Equations with Tree Structures

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**Keywords:** multibody system, equation of motion, iteration methods, simulation efficiency.

In the paper the mathematical methods of computer simulation of multibody dynamics are considered.

Two new iteration algorithms of building-up the equations of motion and their solutions for accelerations without obvious formation of a matrix of system are presented. The algorithm is the modifications of Powell-Broyden's formula variable metrics (the symmetric peer-to-peer formula of count of approximations).

In the first method on each integration step the inverse matrix of a system is improved. This method can be applied to equations of motion with densely filled matrix, for example, for equations of motion in the generalized or quasi -coordinates (in the shape of Lagrange of II form, Euler - Lagrange).

In the second method there is an improvement of a direct matrix of a system. This method can be applied to an equation of motion with banded matrix, for example, for equations of motion in redundant quasi-coordinates (Newton-Euler dynamical equations of rigid bodies, the equations of Lagrange I form).

In the report the properties of iteration algorithms are considered. Their effectiveness is displayed by the examples. It is shown, that the combination of these methods reduce labour-intensiveness of a solution for accelerations of equations of motion for all surveyed mechanical systems.

Efficiency raises due to use of the solutions retrieved on the preceding integration steps in the capacity of initial approximations, and due to reduction of number of iterations required for updating an small perturbations in a matrix of a system.

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# Analytical Method to Analyze Tolerance Effects on Vehicle Ride Comfort

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**Key words:** Tolerance Effect, Sensitivity, Ride Comfort, Weighted RMS, Whole-Body Vibration

## 1. Introduction

Drivers are exposed to the vehicle vibration which mostly originates from the interaction between the road and the vehicle. Vibration can cause discomfort, fatigue, and sometimes even injury to a driver. So, the vehicle ride comfort is one of the most important performance indices to achieve a qualitatively excellent vehicle design. From the previous research works (see, for instance, [1]-[3]), it is obvious that the ride comfort is affected by various parameters of a vehicle model. Therefore, the effects of the parameters on the ride comfort need to be evaluated quantitatively based on the whole-body vibration of the vehicle. An analytical method to analyze the tolerance effects of major vehicle parameters on the vehicle ride comfort is proposed in this paper. The weighted RMS values of the acceleration PSD of a seat position are employed to define the ride comfort.

## 2. Formulation for the tolerance analysis

### 2.1 Sensitivity Equations

Sensitivity equations for a constrained multi-body system can be obtained (see [4] in detail) as follows:

$$\hat{M}\ddot{\mathbf{r}} + \mathbf{\Pi}_r^T \boldsymbol{\mu} = \hat{\mathbf{Q}} \quad (1)$$

$$\mathbf{\Pi} = \left\{ \begin{array}{c} \boldsymbol{\Phi} \\ \boldsymbol{\Phi}_q \mathbf{q}_b + \boldsymbol{\Phi}_b \end{array} \right\} = 0 \quad (2)$$

where  $\mathbf{\Pi}$  represents the composite constraint equations. By employing the above equations, the first order sensitivity can be calculated.

### 2.2 Objective measure of the ride comfort

For whole-body vibration, the frequencies ranging from 0.5Hz to 80Hz are thought to be important. However, since the risk of damage is not uniform for all frequencies, a frequency weighting is used to represent the probability of damage for different frequencies. Human exposure to whole-body vibration can be evaluated using the method defined in [5]. The root mean square vibration magnitude is expressed in terms of the frequency-weighted acceleration which has been used traditionally to describe the vibration at the seat of a seated person. The weighted RMS value is represented as follows:

$$WRMS = \left[ \int_a^b WF^2(f) P^2(f) df \right]^{1/2} \quad (3)$$

where  $WF(f)$  denotes a weighting function,  $P(f)$  denotes a acceleration PSD, and  $a$  and  $b$  denote the lower and the higher frequencies for integration.

## 3. Vehicle modeling and simulation results

Fig. 1 shows a 5-DOF vehicle model undergoing road excitation. This simplified model is devised to simulate bouncing and pitching motion without regarding other behaviors. The vehicle has retained the original sprung and unsprung masses and the pitch moment of inertia. The tolerance effects on the vehicle ride comfort are summarized in Table 1. The standard deviation of weighted RMS with respect to the tolerances of parameters

of a vehicle model is calculated when the tolerances for the parameters are 3.0%. As shown in the table, L4 which denotes the distance between the center of gravity of the vehicle body and the seat position is the most sensitive parameter to the vehicle ride comfort. Fig. 2 shows the standard deviation of the weighted RMS with respect to the amplitude of road profile versus the vehicle speed. As shown in the plot, the standard deviation of WRMS caused by the tolerance of L4 increases as the vehicle speed and the road amplitude increases.

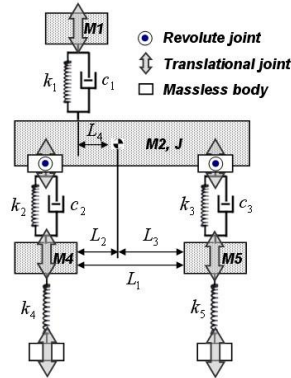
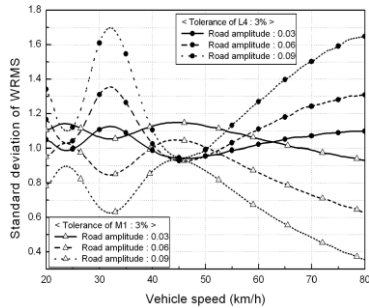


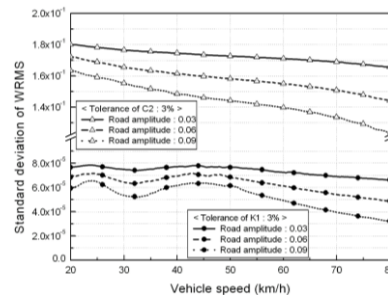
Fig. 1 5-DOF vehicle model

Table 1: tolerance effects of 4 kinds of vehicle parameters

	M	C	K	L
1	<b>6.65 E-01</b>	6.66 E-06	<b>4.99 E-05</b>	3.03 E-05
2	1.07 E-04	<b>1.39 E-01</b>	9.69 E-06	1.73 E-01
3	3.23 E-04	9.81 E-03	1.70 E-06	1.27 E-01
4	4.95 E-04	-	9.57 E-7	<b>9.93 E-01</b>
5	2.84 E-05	-	6.17 E-7	-



(a) tolerance effects of M1 and L4



(b) tolerance effects of C2 and K1

Fig. 2 Standard deviation of WRMS versus the vehicle speed along with the road profile amplitude

## 4. Conclusions

Analytical method to analyze the tolerance effects of major vehicle parameters on the vehicle ride comfort is proposed in this paper and the tolerance effects on the vehicle ride comfort are investigated by employing the method. It is found that the distance between the center of gravity of the vehicle body and the seat position is the most sensitive parameter to the vehicle ride comfort.

## Acknowledgments

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# Real-time multibody vehicle model with bush compliance effect using quasi-static analysis for HILS

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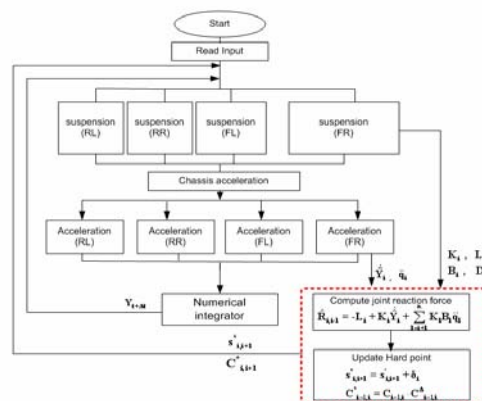
**Keywords:** Real-time multibody vehicle dynamics, Compliance effect

## 1. Introduction

Vehicle handling simulations such as J-turn, slalom, double lane changes, sine with dwell, and fish hook maneuvering are essential ones to test the control subsystems of the intelligent vehicle in the HILS (Hardware In the Loop Simulation). It is well known that the compliance effects from bushes in the suspension subsystem are important in the handling simulations to have reliable solutions. In the multibody vehicle dynamics model, bush elements in the suspension subsystem are modeled as high stiffness linear and torsional springs. However, due to this high stiffness of the bush model, it requires small step-sizes in the numerical integration method. With this kind of bush model, it is almost impossible to achieve real-time simulations in HILS. The quasi-static analysis method which considers bush compliance effects has been introduced firstly by J. H. Lee [1]. Similar concepts are used with Macro-Joint in W. Rulka, E. Pankiewicz [2]. However, both of them are using mass-less link suspension models. So compliance effects are considered along the only longitudinal direction of the suspension link. This paper presents a method to consider compliance effects for the real time multibody vehicle dynamics model based on the subsystem synthesis method[3], using quasi-static analysis. Unlike mass-less link suspension models in previous works, in the subsystem synthesis method, bodies with masses and moments of inertia and joints are used in the suspension subsystem.

## 2. Bush compliance effect using quasi-static analysis

In the multibody vehicle dynamics model, bush elements connect the chassis frame and the suspension linkages in the suspension subsystem. They are usually modeled as high stiffness linear and torsional springs. In order to achieve real-time simulation, compliance effect from the bush element can be considered indirectly using the quasi-static analysis, under the assumption that the gross motion of the vehicle is of interest in the HILS for developing the control subsystems of the intelligent vehicle. In the quasi-static analysis, bush elements in the multibody vehicle model are replaced by corresponding joints. Reaction forces and torques of the corresponding joint are computed. With these reaction forces, torques and the stiffness of the bush element, deformation of the bush elements can then be computed using the quasi-static analysis. According to the deformation, the locations of the joint attachment points (hard points) in the suspension subsystem are corrected to represent compliance effects of the bush elements. Figure 1 shows the computational procedure to update the hard point locations of the suspension subsystem model based on the subsystem synthesis method.

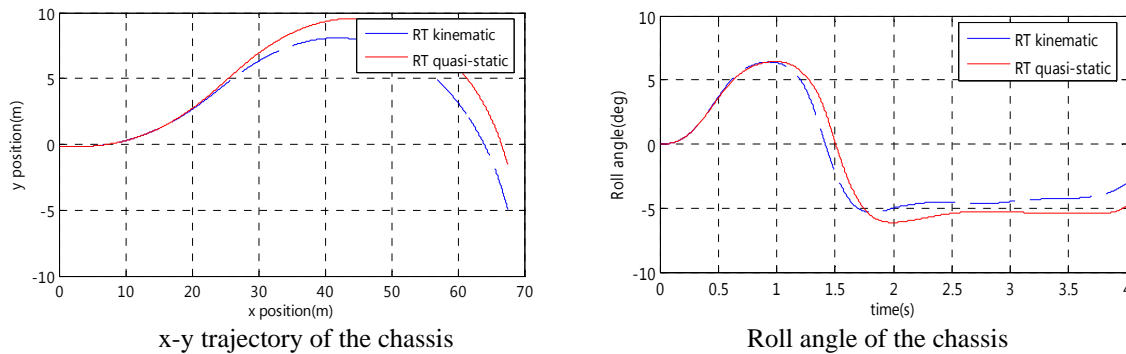


**Figure 1** Data flow diagram of hard point updating process

### 3. Fish hook maneuvering simulation and CPU time measure

A SUV (Sports Utility Vehicle) model has been created. This model consists of McPherson front suspension and Multilink rear suspension system, anti-roll bar with front suspension, and a rack and pinion type steering system. Each suspension system has bump and rebound stoppers. Fish hook maneuvering simulation has been carried whether compliance effects are reasonably considered using the proposed method.

Figure 2 shows the fish hook simulation results. Blue line represents results from the vehicle model without any compliance effects and red line expresses the result from the vehicle with compliance effect. Due to the compliance effects vehicle has larger lateral displacement. Similar trend of simulation results are obtained using ADAMS/car model. Thus, this shows that the proposed method provide compliance effects reasonably.



**Figure 2** Fish hook maneuvering simulation results

CPU time is measured to see the real-time capability of the proposed method. Table 1 shows the CPU time results. Adams bashforth 3<sup>rd</sup> order numerical integration method is used. PC with Intel Core2duo CPU (1.8 GHz) and 2Gb RAM is used to measure the CPU time.

**Table 1** CPU time result

Total CPU time	CPU time / integration step	Total CPU time to Real Time (%)
1.8910	0.00027	27

### 4. Conclusions

Conclusions from this research are as follows;

- 1) The proposed method provides similar trend of solutions to the reference solution from the ADAMS compliance model in the fish hook simulation.
- 2) The proposed method does not spend much CPU time relative to the kinematics model for real time simulation. Thus, the proposed method is well suited for the real time vehicle dynamics model for HILS.

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# Implementation of the Hertz Contact Model on Modelica Language

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**Keywords:** Hertz contact model, Modelica language, Vilke formula, vehicle model, ball bearing model.

## 1. Introduction

It is known [1] to compute a force of the elastic bodies interaction at a contact several different approaches are applied: (a) the classical Hertz model [2], (b) the contact model based on the polygonal approximation of the contacting surfaces [3], implemented on Modelica [4], (c) the volumetric model [1], [5]. In our model we follow the classical Hertz approach, and the normal force computation method is a main topic of our analysis. For definiteness and simplicity to simulate the tangent contact force one uses a regularized model of the Coulomb friction [6]. This is sufficient enough to simulate the dynamics over time of the machine under simulation lifecycle. May be some additional complications for the friction model, e. g. an account of the lubrication of any type, will be needed.

## 2. Computational algorithm

A method to compute the normal force of elasticity for the compliant contact in frame of the Hertz model is pretty standard [7]. An application of the Modelica language [8] makes it easy to accelerate the simulation process significantly if one introduces additional state variables:  $K$ ,  $E$  being the complete elliptic integrals of the first and the second kind respectively, and  $c = k^2$  being the elliptic integral modulus squared, all from the Hertz formulation.

An algorithm based on the known differential equations connecting the variables  $K$  and  $E$  one with another considered as functions of  $c$  [9] and having the following form

$$\frac{dK}{dc} = \frac{1}{2} \frac{E - K(1-c)}{c(1-c)}, \quad \frac{dE}{dc} = \frac{1}{2} \frac{E - K}{c}.$$

In this case the model total DAE system is expanded by an additional ODEs for the variables enumerated above. Thus the elliptic integrals needed are computed concurrently as the state variables while the integration process.

## 3. Simplified algorithm of V. G. Vilke

To compute the normal force under discussion one can apply a formula [5] much more simpler than one of Hertz. This formula is valid for the wide range of eccentricities for the elliptic contact spot. Its expression reads

$$F = 0.3577469 \cdot \frac{2}{3(\theta_1 + \theta_2) A^{3/8} B^{1/8} \sqrt{E(k)}} \delta^{3/2}$$

where  $F$  is the normal force value,  $\delta$  is the undeformed bodies mutual penetration depth,  $\theta_i$  ( $i = 1, 2$ ) are the parameters depending in a known way on the elastic properties of the bodies,  $E(k)$  is the complete elliptic integral of the second kind, its modulus  $k$  is computed via the equation  $k^2 = 1 - A/B$ ,  $A$  and  $B$  are the coefficients in a canonical representation of the quadratic form  $q(x, y) = Ax^2 + By^2$  ( $0 < A \leq B$ ) evaluating the mutual relative penetration depending on the point of the current tangent plane at the contact,  $x, y$  are the coordinates in this plane

Simulations show the Vilke formula relative error is near the value 0.5% of the “exact” value for the normal force derived using the Hertz algorithm. This error holds for the typical applications under evaluation.

## 4. Examples

The proper library of Modelica classes was developed to simulate the 3D multibody dynamics with the contacts of elastic bodies in frame of the known library 3D MBS Dynamics [10], [6], [11]. The following models were created, tested, and analyzed as an examples used for the verification purposes: (a) the vehicle with four

wheels, the skateboard [12], which is able to jump over the road surface, (b) the ball bearings in different dynamic environments.

## 5. Conclusions

The results obtained while developing and evaluating the models under investigation make it possible to conclude that:

1. The Hertz algorithm improved as described above turned out to be even faster than the V. G. Vilke one in the case of the almost circular contact area.
2. The algorithm of V. G. Vilke is more reliable and suitable for wide range of the contact area eccentricities simultaneously providing an accuracy of 0.5% with respect to the Hertz-point algorithm.
3. The Modelica classes developed can be applied to create easily enough 3D dynamic multibody models with contacts for any type of machinery taking into account the Hertz-like contact models.

## 6. Acknowledgements

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# Computer-Aided Models of Freight Cars and Their Applications to Analysis of Some Dynamic Problems

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**Keywords:** simulation of rail vehicles, freight car, multicriteria decision analysis.

Freight cars are usually difficult objects for modeling due to large number of frictional contact interactions. Nevertheless, development of accurate models of cars is very important problem because this type of rail vehicles substantially defines operating costs of railway transport. An additional problem for freight cars with three-peace bogies is the increased risk of derailment. An essential part of researches is devoted to the dynamic analysis of new designed bogies and modification of existing bogies.

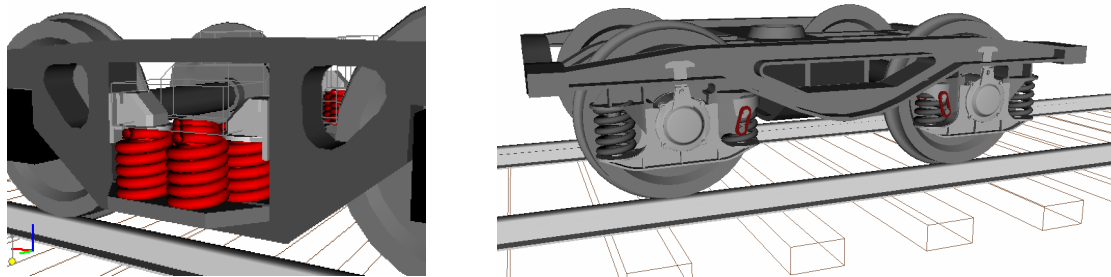


Fig. 1. UM models of tree-peace bogie and Y-25 bogie

Many practical problems can be solved with the help of multivariant parametric analysis for the purpose of the choice of rational parameter values or technical solutions for rail vehicles and track. Such the studies can be formulated and solved as optimization problems; however this method does not include the analysis of the vehicle dynamics features, which is of high importance for design process.

The typical problem statement is as follows. Given are: a set of alternative variants of design and/or parameter values; a set of operation conditions of the analyzed object as subsets of parameters specifying these conditions; a set of variables, which specifies the performance of the object within the given subsets of operation conditions.

The study is formulated as a problem of multicriteria decision analysis taking into account a set of operation conditions of rail vehicles. In the process of solving this problem, influence of alternatives on partial quality factors by specified operation conditions is fulfilled. With this purpose, the hierarchy chart is created for each of the factors on subsets of operation conditions, which allows clearly planning the numeric experiments and analyzing simulation results. Evaluation of an integral factor according to the whole chart gives the most general performance factor, which takes into account all the subsets of conditions involved. Evaluation on a separate branch of the chart is a useful tool for partial analysis. Classical criteria can be used for assessment of the performance factors on the set of operation conditions. In particular, the Bayes-Laplace criterion seems to be the most natural for estimation of the wheel profile wear factor. The minimax criterion can be used for assessment of derailment factor.

It is necessary to separate two types of assessments: on sets of operation conditions and on sets of performance factors. When the assessment on the set of factors is used, a normalization of each of the factors relative to their boundary values is necessary. This normalization could be done by membership function of the factor. This approach allows formalizing fuzzy linguistic assessments like “satisfactory”, “good”, “perfect”, which are often used in normative documents.

The problem of choice of a rational value of the gauge for Russian railways is considered in the paper as an example of the developed approaches. Simulations were performed with the model of a freight car with three-peace bogies, Fig.1.

# Improved efficiency in FFR methods for flexible multibody dynamics by means of shape integrals preprocessing

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**Keywords:** Real-time, flexible multibody dynamics, floating frame of reference.

## 1. Introduction

In flexible multibody dynamics, one of the most widely used methods for considering flexible bodies is the Floating Frame of Reference (FFR) [1]. This method attaches a local frame of reference to each elastic solid, in such a way that the frame undergoes the large rigid body motion, and the elastic displacements are superimposed using local coordinates. The most common way to model the local deformation is by means of the finite element method, so that a reduction procedure, such as component mode synthesis, can be then applied to reduce the number of system coordinates.

In a previous work [2], the authors compare two FFR formulations, one based on absolute natural coordinates [3] and other using relative coordinates, concluding that the absolute method is better suited for small sized problems, whereas the relative method is faster when the system has a large number of coordinates.

In both cases, the co-rotational approximation proposed by Géradin and Cardona [4] is used to obtain the mass matrix and the velocity dependent inertia forces of flexible bodies. These terms are obtained at each time-step by performing matrix products in which the finite element mass matrix is involved. This implies that although the use of component mode synthesis reduces the number of coordinates to be integrated in time, the method does not take full advantage of the reduction, since large matrices of the finite element mode size still appear in the mass matrix and inertia forces calculation.

The main idea of this work is to study the performance of both the absolute and the relative formulations, when these terms are obtained by means of smaller matrix operations, involving matrices of the size of the reduced model. This is achieved by calculating several constant matrices, obtained through shape integrals in a preprocessing stage, as shown in [1]. Once the method is defined, a new comparison between the two formulations will be carried out, in order to check if the performance differences are kept or new criteria should be established.

## 2. Description of the method

The procedure will be described for the absolute formulation, being analogous for the relative one. The position of any given point  $\mathbf{r}$  of a flexible body can be obtained as,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{A}\bar{\mathbf{r}} = \mathbf{r}_0 + \mathbf{A}(\bar{\mathbf{r}}_u + \delta\bar{\mathbf{r}}) \quad (1)$$

where  $\mathbf{r}_0$  stands for the position of the origin of the local frame of reference,  $\mathbf{A}$  is a rotation matrix defined by the three orthogonal unit vectors of the reference frame  $[\mathbf{u}|\mathbf{v}|\mathbf{w}]$ ,  $\bar{\mathbf{r}}_u$  is the undeformed position of the point in local coordinates, and  $\delta\bar{\mathbf{r}}$  is its local elastic displacement. The elastic displacement at a point is approximated using component mode synthesis, which can be written in matrix form,

$$\delta\bar{\mathbf{r}} = \mathbf{X}\mathbf{y} \quad (2)$$

being  $\mathbf{X}$  a matrix containing the mode shapes as columns, and  $\mathbf{y}$  the vector of modal amplitudes. The body motion can be defined by the vector of variables of the body  $\mathbf{q}$ , which contains the position of the origin  $\mathbf{r}_0$ , the local frame vectors  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{w}$ , and the modal amplitudes  $\mathbf{y}$ ,

$$\mathbf{q}^T = \left\{ \mathbf{r}_0^T \quad \mathbf{u}^T \quad \mathbf{v}^T \quad \mathbf{w}^T \quad \mathbf{y}^T \right\} \quad (3)$$

By time differentiation of (1), a linear relationship can be established between the velocity of a point  $\dot{\mathbf{r}}$  and the time derivatives of the body coordinates  $\dot{\mathbf{q}}$ ,



$$\dot{\mathbf{r}} = [\mathbf{I}_3 \quad \bar{x}\mathbf{I}_3 \quad \bar{y}\mathbf{I}_3 \quad \bar{z}\mathbf{I}_3 \quad \mathbf{AX}] \dot{\mathbf{q}} = \mathbf{B}\dot{\mathbf{q}} \quad (4)$$

where  $\bar{x}$ ,  $\bar{y}$  and  $\bar{z}$  are the components of the deformed local position of the point  $\bar{\mathbf{r}}$ . This relationship can be introduced into the kinetic energy expression, to obtain the mass matrix  $\mathbf{M}$ ,

$$T = \frac{1}{2} \int_V \dot{\mathbf{r}}^T \dot{\mathbf{r}} \, dm = \frac{1}{2} \int_V \dot{\mathbf{q}}^T \mathbf{B}^T \mathbf{B} \dot{\mathbf{q}} \, dm \Rightarrow \mathbf{M} = \int_V \mathbf{B}^T \mathbf{B} \, dm \quad (5)$$

which, after developing the matrix product  $\mathbf{B}^T \mathbf{B}$ , has the following form,

$$\mathbf{M} = \int_V \begin{bmatrix} \mathbf{I}_3 & \bar{x}\mathbf{I}_3 & \bar{y}\mathbf{I}_3 & \bar{z}\mathbf{I}_3 & \mathbf{AX} \\ \bar{x}^2\mathbf{I}_3 & \bar{x}\bar{y}\mathbf{I}_3 & \bar{x}\bar{z}\mathbf{I}_3 & \bar{x}\mathbf{AX} & \\ & \bar{y}^2\mathbf{I}_3 & \bar{y}\bar{z}\mathbf{I}_3 & \bar{y}\mathbf{AX} & \\ & sym. & \bar{z}^2\mathbf{I}_3 & \bar{z}\mathbf{AX} & \\ & & & & \mathbf{X}^T \mathbf{X} \end{bmatrix} dm \quad (6)$$

where all the terms can be manipulated to put the body coordinates  $\mathbf{A}$  and  $\mathbf{y}$  outside the integrals. For example, the integral of  $\bar{x}$ ,

$$\int_V \bar{x} dm = \int_V (\bar{x}_u + \mathbf{X}_x \mathbf{y}) dm = m\bar{x}_u^G + \left( \int_V \mathbf{X}_x dm \right) \mathbf{y} = m\bar{x}^G \quad (7)$$

In this expression,  $\bar{x}_u$  is the undeformed local  $x$  coordinate,  $\mathbf{X}_x$  is the first row of  $\mathbf{X}$ ,  $m$  is the mass of the solid, and  $\bar{x}_u^G$ ,  $\bar{x}^G$  are the local  $x$  coordinates of the center of mass in the undeformed and deformed configurations respectively.

A similar procedure can be applied to all the remaining terms. All of them can be obtained from matrix operations involving the modal amplitudes  $\mathbf{y}$ , and constant matrices resulting from integrals of the mode shapes, the undeformed positions, and products between them. In the case of the relative formulation, the  $\mathbf{B}$  matrix relates different coordinates to the nodal velocities, and the expressions are somewhat more complicated, but the procedure for obtaining the mass matrix is analogous.

As it was done in [2], a comparison between both formulations will be carried out by simulating different systems, such as the Iltis vehicle running through a rough profile (Fig. 1), and criteria will be established to decide which formulation should be used depending on the problem size and the number of elastic bodies.



Fig. 1. Iltis maneuver.

Preliminary tests have shown that, in the absolute method, with flexible bodies modeled using 10 elements (33 DOFs) and 6 modes, performance can be increased about 175% by using preprocessing. In the case of large finite element models, the difference can be much more significant, since preprocessing makes the CPU-time independent of the finite element mesh size.

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# Stress load and durability analysis using multibody approach

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**Keywords:** durability analysis, fatigue, hybrid mechanical systems.

## 1. Introduction

The present paper describes the CAE-based approach for durability analysis that is being implemented in Universal Mechanism software to predict the fatigue damage of parts of mechanical systems. The approach predicts fatigue strength of structural components of machines and mechanisms based on results of simulation their dynamics taking into account real working conditions.

## 2. Durability analysis

The analysis starts with the dynamical hybrid model in Universal Mechanism. The flexibility characteristics of the structural parts are incorporated into UM model using a modal formulation based upon component mode synthesis. Basically, this method represents the part's flexibility using a modal basis, which is optimized to account for constraint and force locations. The mode shape displacements and stresses are calculated using the finite element programs ANSYS or MSC.NASTRAN.

The durability analysis combines the stress time history information generated during series of numerical experiments in UM and the material fatigue strength characteristics to generate the predicted life distribution in the part.

Any durability analysis relies on three key inputs: stress loading data - time history of the stresses, material data that describes how the material reacts to repeated stress application at various stress levels and parameters of durability calculation method.

By employing the full finite element representation of the component in the UM model, the local stresses are directly obtained as result of the UM solution. Flexible body deformations are modeled as a linear combination of mode shapes. As long as the number of mode shapes selected adequately the modal superposition will model deformations accurately and efficiently.

The idea that the deformation of a flexible body can be represented by the sum of a number of mode shapes, scaled by appropriate factors, can be extended to stresses in the body as well. These factors, or modal coordinates, can be used as the scaling factors on the stress solution of each mode shape and the superposition of these scaled stresses represents the body's instantaneous stress state. If the superposition is performed at every node in the finite element model, for every time step in the UM solution, the stress time history is defined at every location.

The modal coordinate time history can be saved for every numerical simulation. Based on this modal time history and file with orthonormalized mode shapes the stresses at every node can be obtained. When the durability analysis is started, the user is prompted for the location of modal coordinate time history files and orthonormalized mode shapes files, then the type of analysis required and the material data to be used.

With all of the parameters set, durability analysis in UM performs the stress at every node, and then proceeds to multi channel peak/valley extraction and rain flow cycle counting, followed by the damage sum.

## 3. Applications

The described approach is implemented in Universal Mechanism software. Estimation of stress load and fatigue strength of long-wheelbase flat-cars of a new type and a bogie frame of a locomotive is discussed.

## 4. Acknowledgements

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# Modeling and Simulation of the Dynamics of Flexible Composites

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**Keywords:** Dynamics of Flexible Systems, Modeling of Composites, Simulations

The paper considers flexible composites as a new generation systems with important applications in high technologies. The flexible systems have attracted a great deal of attention in the last few decades due to the potential benefits they offer over the conventional systems in various applications. Dealing with active systems requires the possibility of modelling and simulation of their dynamical behaviour. Because of their heterogeneous nature the modeling comprises the coupling effects of multiple simultaneous physical phenomena and involves the solving of coupled systems of partial differential equations.

The exact solutions for the dynamics of such systems can be obtained only on very strong restrictions. The approach to present the light-weight high-performance flexible structures like mechatronic systems is very helpful. Finite element methods (FEM) are a prospective approach for investigating the dynamics of such systems.

This paper presents an advanced investigation in modelling of the dynamics of flexible composites. A computer-based model for the dynamics of the considered flexible systems is developed. Because of the large systems of equations, the objective is to create a numerically effective finite element tool and a new approach in simulation and analysis of the behaviour of these systems. The main aspects of the application of the model are discussed through a set of numerical examples. The simulations present more precisely the real dynamics of these systems.

# Comparison of two thick plate–elements based on the absolute nodal coordinate formulation

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**Keywords:** plate element, multibody dynamics, finite element method, eigenfrequencies.

## Abstract

Two formulations for a flexible quadrilateral thick plate element for dynamic analysis within a multibody dynamics environment are compared: a fully parameterized element and a fully parameterized element with linearized transverse shear angles to overcome shear locking. The elements are derived within the absolute nodal coordinate formulation (ANCF) and can undergo large displacements, large rotations and large deformations.

The standard fully–parametrized plate element has degrees of freedom for the deformation of the cross section and consequently will exhibit additional high frequencies. This is one of the reasons for the slow convergence of the standard plate element. The linearized shear approach could also resolve these problems. In earlier studies it has been shown that Poisson locking can be avoided by using simplified material models. Therefore, in this study also two material models are considered: a linear elastic isotropic model and one where the Poisson effect is neglected.

The presented elements are benchmarked to analytical solutions in a linear static analysis and in an eigenvalue analysis. In addition, the accuracy of reduced numerical integration of the elastic forces within the element is considered. Finally, the capability of undergoing large rotations is tested by means of a simple static test for both elements.

The plate–element with the linearized shear angles gives accurate results under shear deformation loading. In the case of pure bending and zero Poisson ratio, both elements give correct results in the small deformation case. However, for the standard element with a linear isotropic elastic material the Poisson locking occurs and the convergence is poor. In addition, the small displacement static test shows that the use of the simplified material model leads to over-soft behavior. The same effect can be observed in the eigenvalue analysis through shifting of eigenfrequencies and eigenmodes.

In the large rotation test the standard element fails, whereas the linearized shear element gives accurate results. The main reason for the failure is the curvature or thickness locking.

The reduced numerical integration of the elastic forces within the element works well for the small deformation case. However, in the case of large deformations a larger number of integration points is necessary to obtain sufficient accuracy.

# Hysteresis Modeling in Electro-Magneto-Mechanical Systems

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**Keywords:** Electro-Magneto-Mechanics, Domain Interaction, Magnetoelectricity, Piezoelectricity, Hysteresis, Jiles-Atherton Model

## Abstract

Mechatronic systems, by their very meaning, comprise (at least) mechanical and electromagnetic subsystems. Moreover, a characteristic feature of these systems is the non-linear interaction between the physical domains. The correct representation of the interaction is a vital point in the dynamics simulation of mechatronic systems. This presentation pursues a unified modeling of electromagnetic networks and of electromagnetomechanical interactions. We derive the motion equations of electromagnetomechanical systems (EMMS) synthetically as well as from a Lagrange model. The Lagrangian motion equations naturally account for nonlinear interactions. Special attention is given to the piezoelectric and piezomagnetic effects.

A way to model electromechanical systems is the charge approach, i.e. the configuration of the electrical subsystem is represented by the charges in the fundamental loop of the electrical network graph [1, 4, 5]. We present an extension of this concept to EMMS, where the flux linkages in the fundamental loops of the magnetic network graph constitute the generalized coordinates of the magnetic subsystem.

An important point in EMMS dynamics simulation is the consideration of non-linear characteristics within the electromagnetic network, in particular saturation and hysteresis phenomena. This presentation focuses on the modeling of hysteresis effects in ferromagnetic and ferroelectric materials. The underlying hysteresis model is the Jiles-Atherton (JA) model [2, 3, 7] for ferromagnetic material, which attains the form

$$\frac{dM}{dH} = f(M, H), \quad (1)$$

where  $M$  is the magnetization density and  $H$  the magnetic field strength. In contrast to the classical Preisach-model, the JA-model is rooted in physical considerations. An analog model holds for ferroelectric materials [6], replacing  $H, B$ , and  $M$  with  $E, D$ , and  $P$ , respectively, where  $E$  is the electric field strength,  $D$  the dielectric displacement, and  $P$  the electrical polarization density. A model of the form (1) can, however, not be employed within the flux approach, i.e. with flux linkages as generalized coordinates. We therefore derive an alternative formulation of the form

$$\frac{dM}{dB} = f(M, B), \quad (2)$$

where  $B$  is the magnetic flux density, related to the field strength by  $B = \mu_0(H + M)$ . We point out the numerical properties of this formulation, as far as they are relevant for the numerical simulation. Notice, that the magnetization, respectively the electrical polarization, are internal state variables. Thus, the generalized coordinates of the electromagnetic subsystem comprise the fundamental loop charges, the fundamental loop fluxes, and the magnetizations and electrical polarizations.

Finally, a systematic approach for EMMS modeling is presented, that builds upon magnetic and electric two-pole primitives. A matrix formulation is presented, that can be implemented straightforwardly. Some example of practical relevance conclude the presentation.

## Acknowledgment

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# Optimum Design of a Mechanism using a Multibody Model and Response Surface Analysis

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**Keywords:** Central composite design, Design of Experiments (D.O.E), Response surface analysis, Sequential Quadratic-Programming (SQP) method, Analysis of variance

## 1. Introduction

As the production cycle of goods gets shorter, analysis and designs using a computer model become all the more important. A simulation model with reliability reduces designing time and improves efficiency by being applied to guide the design direction of a real system. This paper presents an optimal design method using design of experiments (D.O.E). D.O.E is the method to guide the order and number of experiments to get the desired information from minimum experiments. Before an optimization is operated, Plackett-Burman design, one of the factorial design methods, is used to choose the design variables which affects a response function significantly[2]. Then, experiments are made according to a central composite design table. Next, using the response surface analysis, second order recursive model function, which informs a relation between design variables and response function, is estimated [1]. In order to verify the reliability of the model function, analysis of variances(ANOVA) table is used. Sequential Quadratic-Programming (SQP) method is used to find the value of design variables which minimize the model function and satisfy the constraint conditions[3]. As applying the above procedure to a multibody dynamic model and comparing the values of response functions of a current and optimized system, the usefulness of the optimal design method presented in this paper was verified.

## 2. Main body

Figure.1 shows the procedure to make a design optimization using D.O.E. First, a characteristic value, design variables and constraints should be defined. The characteristic value means a response value of a system that a designer want to make maximized or minimized. When defining design variables, the level of variables should be defined. The level is the number of values that variables can take. For example, a 2 level variable can have only maximum and minimum values, but a 3 level variable can have minimum, maximum and neutral values.

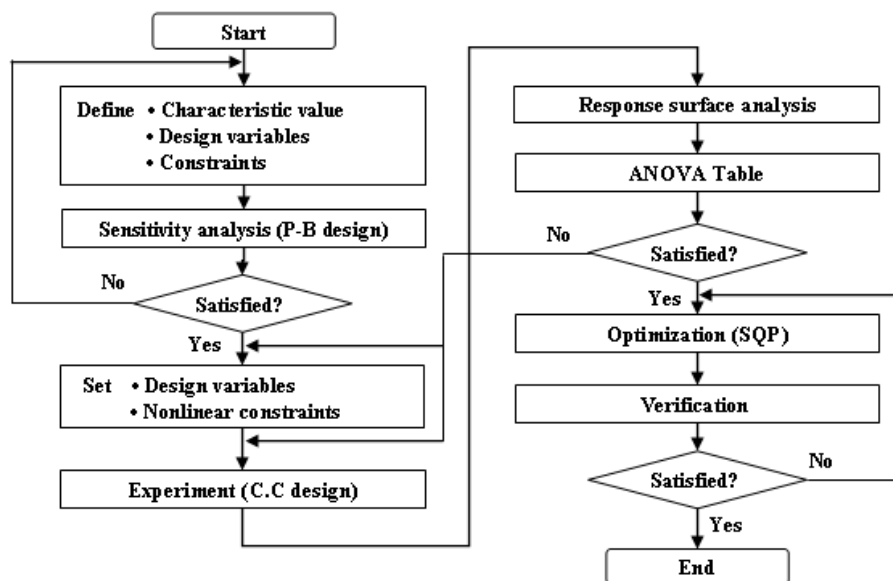


Figure 1: Optimization procedure

Among many design variables, in order to select the variables which affect the change of the characteristic value significantly, sensitivity analysis is performed. This paper uses Plackett-Burman design, one of the sensitivity analysis methods using the 2 level factor design, which reduces the number of experiments considerably. After selecting design variables to be optimized, the relation between the characteristic value and design variables should be defined as a function. Response surface analysis uses a second order recursive model to estimate the function, assuming that the function is nonlinear. In order to find the recursive model function through a small number of experiments, the central composite design table, the orthogonal array which adds the central and axis point to the 2 level factor experiment, is used. The presumed model function is verified by the ANOVA table [1]. Finally, the value of design variables which minimize the model function and satisfy the constraint conditions should be found. In this research, SQP method, which minimizes the objective function that consists of N variables efficiently and satisfies the nonlinear constraint conditions, is used as an optimization algorithm [3].

By using the above optimization process, the geometry of the rolling crank and the shape of the cam are optimized to increase the cutoff velocity of the machine to shut off the extra high voltage. The cam is operated when the spring compressed by an electric motor is relaxed. The crank connected with the cam interrupts an electric circuit by rotating a constant angle. To improve the cutoff velocity, the energy loss, which occurred when the potential energy of a spring is converted into the kinetic energy of a crank, should be minimized. Thus, to maximize the power transferred from a cam to a crank, the minimization of the pressure angle is set as an objective function. Table.2 shows optimization results. After the optimization, the average pressure angle is down about 17.9 percent

Table 1: Optimization results

Variables	Initial	Optimized
V1	94.0 (mm)	91.0 (mm)
V2	135.0 (mm)	133.0 (mm)
V3	17.5 (mm)	23.5 (mm)
V4	117.5 (mm)	103.5 (mm)
Average Pressure Angle	34.3801 (deg.)	28.2235 (deg.)

### 3. Conclusions

In this research, the optimum design method, which minimizes the characteristic value of a system using response surface analysis, is presented. To verify the reliability of the optimization procedure, a multibody model of electric circuit breaker is constructed using a commercial multibody dynamics program. After optimization, the value of an objective function is decreased to 17.9 percent. In addition, a real system composed of optimized design variables is tested and the test results are the same with simulation results. Therefore, the optimization of the system has been done successfully and also the optimization result is verified.

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# Simulation of motion of rover with elastic wheels

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**Keywords:** simulation, mechatronic systems, wheeled robots, flexible MBS.

## 1. Introduction

The task on modeling a rover on elastic wheels includes development and construction of a model, analysis of its properties at movement on a ground, comparison with the model of rigid non-elastic wheels, determination of its advantages and lacks. In this paper we elaborate a wheel model as a multibody system in accordance with the multibody dynamics simulation paradigm.

## 2. Description of the model of a wheel and rover

Let us consider a pneumatic wheel which consists of a disk and a tire. The disk is represented by a rigid body, the tire is elastic. For modeling the tire we shall present it as a ring chain of rigid bodies connected by elastic elements. For elastic communication of the tire with the disk we shall also connect them by elastic force elements. The general view of model of the wheel is given in Fig 1.

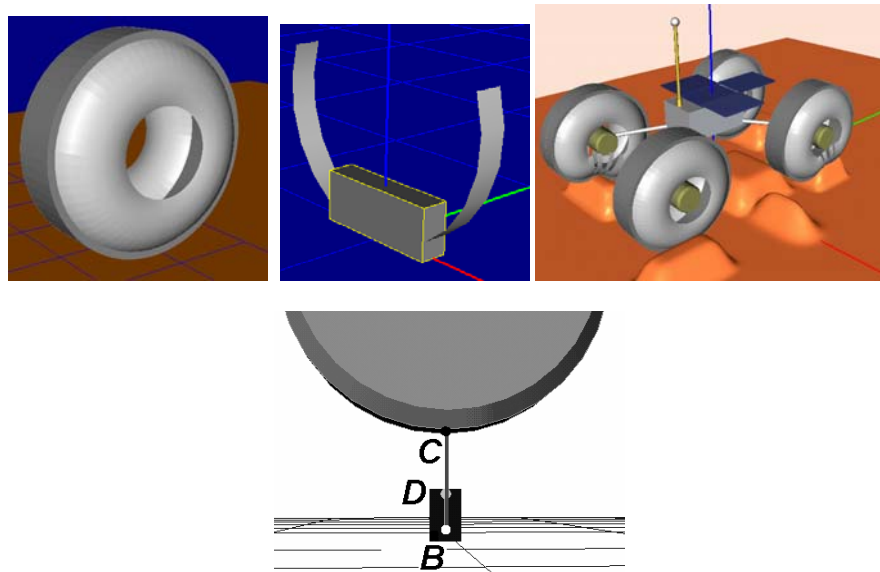


Fig 1. Model of a wheel with the tire containing 64 elements, a separate element of the tire and rover.  
*Points of force application.*

One element of the tire is a rigid body (Fig. 1, center). Two neighbor elements are connected by a rotary hinge. The last (64-th) element is connected both with the 63-th one and with the 1-st element closing the chain.

The rigid disk in the wheel model transfers torques and force influences to the tire. The disk is connected with the tire only by force elements. Force elements have attachment points on the disk (B) and on the tire (D). The scheme of these connections is shown in Fig. 1. Action of this force between its initial and end points is equivalent to a preliminary stretched spring. That is it attracts points. According to an arrangement of points it extends the tire. The same scheme allows transferring the torque from the disk to the tire effectively.

For modeling movement of the system on a surface, contact forces of regular type are used: they realize contact of a "points-surfaces" type. It is necessary to use up to 8 points of contact for each element of the tire. The chosen scheme is sufficient for modeling the motion across a step or other obstacles. Division of contact

forces into two sets is caused by the necessity to have different parameters for these forces. It realizes also an opportunity to change easily positions of points and it is used for modeling of grousers on a wheel as well.

The adjustment of model parameters was made to make it useful for simulation of a pneumatic wheel. Initial data for adjustment are data about the volume and areas of contact of a pneumatic wheel at its deformation while the clearance of a rover is changed. By preparation of these data it was supposed that the tire is deformed by a plane, thus the resulting volume and the area of a spot of contact were calculated.

After a series of experiments on deformation model of a wheel (with lateral displacement) obtained data on conformity of value of force (between a disk of a wheel and a separate element of the tire) and an average (on a wheel) and the lateral displacement were approximated by function (1).

$$F = \frac{c_1}{(dz + dy)^{c_2} + c_3 \cdot (dz + dy)^{c_4} \cdot dx^{c_5} + c_6 \cdot dx^{c_7}} + c_8, \quad dx = \left( \sum_{i=1}^{64} x_i \right) - 64 \cdot x_0 \quad (1)$$

where  $dx$ ,  $dy$ ,  $dz$  are average deviations on base axes.

On the basis of constructed model of a wheel the computer-aided model of a four-wheel rover were built. The model of a wheel adjusted at the previous stage is used as a base primitive for the description of models of all rover wheels. The rover case is a rigid body. The case and disks of wheels are connected by levers with rotary hinges on ends. Driving torques in hinges simulate action of engines. Totally the rover model has 261 body (260 for modeling wheels), 286 degrees of freedom, 1029 internal force elements, 512 contact forces (Fig 1). Longitudinal base is 2.1 m, cross-section base is 1.6 m, weight of the case is 235 kg, rover weight is 274.56 kg, and height of the case above the ground is 0.8 m.

Contact to a surface is defined by contact stiffness (600 N/m), additional environments factor (100), the static coefficient of friction is 0.5, and the kinematic one is 0.45. For modeling the ground resistance at movement of the wheels, additional forces acting on elements of the tire are introduced. Forces act on elements of the tire pressing through a ground "downwards-forward" and are directed against their speed of motion. The forces act in such a manner that the factor of motion resistance is close to 0.05.

A series of simulation experiments were done. Main experiments were devoted to analysis of the rover motion on a surface with stones and on slopes. Summary of simulation results for rover motion along the surface "with stones" is shown in Table 1. Different sizes (heights) of obstacles were taken into account.

Table 1

$h$	$V_{max}$	$V_{min}$	$V_{aver}$	$Vy_{max}$	$Vy_{min}$	$Vy_{aver}$	$Om_{max}$	$Om_{min}$	$Om_{aver}$
0.146	0.123	0.059	0.08195	0.1215	0.0589	0.0812	0.2007	0.0918	0.1341
0.223	0.1283	0.0491	0.08099	0.1281	0.0483	0.0801	0.2063	0.0648	0.1273
0.323	0.268	0.0411	0.0845	0.2089	0.0122	0.0789	0.3185	0.0221	0.1327

### 3. Conclusion

Modeling has shown that elastic wheels perform better on complex, non-flat surfaces (stones, slopes, sand soil with slippage, etc), providing a smoother movement of the rover chassis. More precisely, these results mean the following. Modeling has shown basic practicability of rover motion on large inflatable wheels on rough country. The key parameters of movement (oscillations of the case, an opportunity of overcoming larger obstacles, etc.) have appeared to be better than for similar machines on rigid wheels.

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# Real-time simulation of parametric road vehicle models

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**Keywords:** simulation of road vehicles, real-time simulation, parametric car models

## 1. Introduction

The dynamical analysis of all kinds of vehicles is a well established approach based on highly developed software tools for modelling, simulation and animation of the resulting motions. However, usually the vehicles are considered as open loop systems without sophisticated driver models. On the other hand, driving simulator laboratories were in use in many places where the driver is operating a virtual vehicle with real time dynamics, see e.g. [1]. For the assessment of road vehicles the steering performance is an essential feature. Therefore, it is proposed to complement automobile models at personal computers by human drivers operating a steering wheel and pedals, and providing the motion in real time at the screen of the computer used. To meet the real time requirement reduced car models are used provided by commercial or research software in a fully parametric manner.

## 2. Car model, subsystem modeling, track implementation

The mechanical parts include the vehicle body and four wheels interconnected by the kinematics and dynamics of the suspension as well as the engine dynamics and the tyre characteristics. All these subsystems and the corresponding data are put in a MATLAB-SIMULINK environment supporting a hierarchical modelling.

The open-loop system is controlled by steering angle and pedal positions operated by a human driver observing the vehicle motion via the computer screen. The dynamics of the closed-loop system can be systematically investigated and the parameters of the model serve as design variables for optimization purposes. In addition, the dynamical behaviour of human drivers may be identified and the vehicle parameters may be adapted resulting in personal car dynamics.

The challenge consists in dealing with the following problems as discussed in [2].

- Fast processing to obtain real time simulation,
- Complex modelling for human driving,
- Avoiding the numerical singularities,
- Input signals rendered suitable for simulation,
- Modelling all subsystems capable to give the driver a real feedback sensation,
- Solve the loops inside the model,
- Reproduce the specific characteristics of a F1 race car in the model and in the tests.

The subsystem modelling has been optimized to reach a good response with an extremely low computational cost. The implementation of external peripheral devices gives the opportunity to introduce real signals in other kind of simulations, not only related to vehicle dynamics. The Virtual Reality Interface is a fundamental part and has been developed on purpose for it, overcoming several problems related to the implementation process.

The implementation of the full track of the Barcelona Montmelò circuit provides a real driving experience extremely useful for the technical analysis and gives the drivers the feeling of a real track. Thus, this paper highlights the possibilities of the simulator to preview car behaviour on different manoeuvres, bringing the car to its limit. Moreover, the comparison between the NEWEUL model and the SIMPACK models underlines the differences in models structuring. In particular, the importance of the tyre model appears fundamental to obtain suitable and reliable results on the car behaviour, especially when transient manoeuvres are frequent. Finally the live visualization of the car behaviour turns out to be a powerful instrument for intuitive analysis to understand and solve programming and debugging problems. Using optimization techniques as presented in [3], an optimal test driver can be modelled for a given track.

### 3. Conclusions

The paper will report on the detailed modelling of a F1 race car, Figure 1, the simulation and animation aspects of MATLAB-SIMULINK and a comparison of the performance of a driver on some typical race tracks. It will be shown how a driver is adapting to different tracks representing their training capability. The driver may also be replaced by a driver model the development of which was recently reviewed in [4]. Then, real-time simulation is still an important issue.



Figure 1: F1 race car model

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# A linear formulation for multibody dynamics simulation with unilateral constraints and Coulomb friction

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**Keywords:** multibody, simulation, joints, friction, unilateral.

An algorithm for solving the forward dynamics of a multibody system with bilateral constraints is introduced. A further extension allowing systems with unilateral constraints and dry friction is given, while preserving generally the algorithms framework. In order to solve the system for unknown reaction factors a package for solving sparse linear equation systems is used for the most part.

For clarity sake the constraints imposed on the system are considered to be represented with d6 joints with limits. That allows involving into consideration a wide selection of joints, including those with friction and unilateral, maintaining uniformity of presentation. Other types of constraints can be reviewed similarly. D6 joints with limits have a Boolean flag variable for each limit reached, denoting whether the joint is in state of adhesion or separation for that limit. Joints with friction also have a flag indicating whether the joint is in motion (slipping, spinning or both) or static.

We may try to remove separating unilateral constraints from the system, and treat adhesive ones as bilateral. Furthermore jamming static joints with friction (the d6 joints closes on all axes), and treating dynamic friction in moving joints as an active force allows presenting them as bilateral too. Thus if the set of state flags in each joint is correctly defined, a system with unilateral d6 joints and friction can be described as a system of bilateral d6 joints at any single time point. Thus many constraint systems can be modeled as time-pieceswise bilateral, if switching points are infrequent enough for that to be computationally effective.

The algorithm encounters three major problems:

- Preliminary detection of a switching point
- Solving switching point, finding the right set of joint states
- Friction model for joints in motion

Some switching points like those caused by impact or sticking must be predicted in advance, because poor timing may lead to energy loss or incorrect system behavior. The simulation step must be correctly adjusted to end at the switching point.

Solving switching point can start from simple guessing. Changing state of the joint where inconsistency occurred may solve the problem. For example separating unilateral joint which is no more adhesive may often give the right solution. However, in a complex case the whole set of system joint state flags must be newly defined. For  $n$  flags in the system that gives  $2^n$  variants of solution, what can be too much to overhaul. An alternative method can be used, for example reformulating dynamics problem as a linear complementarity problem (see for example [1]), which is designed for similar cases and can evaluate the correct set of flags. Solving linear complementarity problem is slow in comparison to linear equations, but considered to be applied only at a switching point in overall it can be effective.

An iterative method varying the right side of equations system, while matrix decomposition in the left side is kept constant, is used to determine friction forces for joints in motion.

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# Motion equations of a 6-dof parallel mechanism with differential constrains

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**Keywords:** parallel mechanism, differential constrains, composed rigid body algorithm, Kane equations, calculation efficiency

## 1. Introduction

Motion equations of a specialized controlled mechanism are considered. This mechanism provides a preliminary mechanical connection (capture), an attenuation off kinetic energy, an alignment and a retraction of two free flight objects moving one relatively another. Installed on the one of this objects it has a connecting ring associated along its perimeter with a structure base by means of six parallel guiding kinematical chains, i.e. it is a parallel mechanism [1] with six degrees of freedom (6-dof). Differential constrains between guiding kinematical chain and theirs separate links provide high efficiency of the mechanism during execution of mentioned above objects joining stages. Accelerations, velocities and positions of the mechanism are calculated most effective because of closed form of motion equations with minimal dimensions and a symbolic realization of calculating algorithm.

## 2. Mechanism kinematics and motion equations features

Each guiding kinematical chain consists of one screw, one screw-nut joint and two universal joints, connecting the screw with the ring and the mechanism structure base. One screw-nut joint has two degrees of freedom because of the nut is movable. Its kinematical constrains equations bind differences of screw and nut displacements, velocities and accelerations. These constrains named "differential" are similar to differential gear constrains. The guiding kinematical chains are connected in pairs in the following way. Both screws in each pair are joined by conical gears with a common axle and may rotate only in the same direction. Both nuts in the same pair are combined in a similar manner. Six screw linear displacements correspond to linear and angular movements of the ring. They are transformed by screw-nut joints to screw and nut pairs rotations depending on acting loads. Screw pair rotations correspond to a difference of screw pair linear displacements, that is to lateral displacements and roll rotations of the ring. Nut pair rotations correspond to an average of screw pair linear displacements, i.e. to an axial displacement and yaw-pitch rotations of the ring. So a constrains structure of the guiding kinematical chain pairs separates 6-dof ring motion on two different component groups having different kinetic energy during the objects joining and damped by using devices with different attenuation capacity. Moreover rotations of nut pair common axles are passed by torsion springs to three axes of a special device of three differential gears. Fourth axis of this device is joined with a friction clutch and an electrical drive of the mechanism. This combination of three differential gears has three own degrees of freedom. It allows redistribute guiding chain pairs motions depending on moments acting on four device axes and provides particularly additional yaw-pith ring rotations remissive capture time, a separation of high energy axial motion of the ring for its attenuation by using the friction clutch and also a redistribution of an electrical drive moment during alignments and retractions of the ring.

Motion equations are formed by using relative motion variables therefore mechanisms constrains are transformed to a tree structure. All the joints connecting the ring and guiding chains, the screw-nut joints and also the screws and nuts connecting gears are replaced by  $m = 36$  kinematical constrain equations relative to joint coordinates, velocities and accelerations. Thus transformed tree structure mechanical system has  $n = 42$  degrees of freedom, i.e. a generalized mass matrix  $\mathbf{A}$ , a joint relative accelerations vector  $\ddot{\mathbf{x}}$  and a generalized force vector  $\mathbf{b}$  have dimensions of  $(42 \times 42)$  and  $(42 \times 1)$  respectively in closed form dynamic equations

$$\mathbf{A} \ddot{\mathbf{x}} = \mathbf{b} . \quad (1)$$

The generalized mass matrix  $\mathbf{A}$  is block-diagonal because of existing of parallel kinematical chains. The ring in such transformed system is described as a free body, connecting with the structure base by a fictitious 6-dof joint. A sub-matrix  $\mathbf{A}_R$  and a sub-vector  $\mathbf{b}_R$  correspond to ring motion. A recursive composed rigid body algorithm [2, 3] provides calculations of generalized mass a sub-matrix  $\mathbf{A}_i$  and a generalized force

sub-vector  $\mathbf{b}_i$  corresponded to the guiding chain  $i$  ( $i=1, \overline{6}$ ) with  $O(n^2)$  efficiency where  $n$  is chain length. Diagonal elements of  $\mathbf{A}$  and elements of  $\mathbf{b}$  for gear links are determined by rotation motion equations. Kinematical loop constrain equations written relative to positions  $\mathbf{x}$ , velocities  $\dot{\mathbf{x}}$  and accelerations  $\ddot{\mathbf{x}}$  vectors are free from singularity. Their solutions by generalized coordinate partitioning [4] are used for a dimension reduction of dynamic equations of the tree structure mechanical system. There are three levels of constrains. Screw and nut connecting gear constrains and also screw-nut joints differential constrains of separate pairs of guiding kinematical chains produce first and second levels accordingly. "Ring-guiding chain" constrains produce the third level. Solutions of constrain equations for positions, velocities and accelerations of joint relative motions are obtained from the lowest to the highest level. Inversions of only three  $(2 \times 2)$ -matrixes for the second and six  $(3 \times 3)$ -matrixes for the third constrain level are needed at that. Expressions of dependent accelerations through independent one allow step by step reduce dimensions of parallel mechanism dynamic equations. Finally a modified generalized mass matrix  $\mathbf{A}^* = \mathbf{A}_R^*$  and a modified generalized force vector  $\mathbf{b}^* = \mathbf{b}_R^*$  of (1) have  $(6 \times 6)$  и  $(6 \times 1)$  dimensions accordingly. This procedure increases a simulation efficiency because of  $n - m \ll n < n + m$ . Equations of generalized reactions in replaced "ring-guiding chain" joints are solved after calculation of an independent accelerations vector  $\ddot{\mathbf{x}}^* = \ddot{\mathbf{x}}_R$  of the ring. Generalized reactions are recalculated to the Cartesian forces and moments, acting on the structure base of the parallel mechanism.

The combination of three gear differentials joined with three pairs of guiding kinematical chains is described by closed form motion equations obtained by using Kane equations [5]. Its kinematical loop constrain equations are free from singularities. Their solutions by using generalized coordinate partitioning provides reduction of dynamic equations so that an inertia matrix has finally  $(3 \times 3)$ -dimension. These dynamic equations have constant and therefore once calculated coefficients because of permanent spatial orientation of differentials wheels axes.

A stiffness of differential motion equations is decreased because of growing values of generalized matrix elements after generalized partitioning and equations dimensions reduction. A symbolic realization of sparse matrix operations removes redundant arithmetical instructions from expressions for all calculations.

### 3. Conclusions

The closed form of motion equations with minimal dimensions and stiffness, equations coefficients calculating algorithms provide more effective simulation of the above described multi loop parallel mechanism, than all well known recursive motion equations, particularly an articulated body algorithm for multi loops system [6].

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# Damping models for multibody dynamic simulations

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**Keywords:** Linear damping, quadratic damping, Coulomb damping, stick transition velocity.

## 1. Introduction

To decrease vibration and noise in mechanical systems, dampers are necessary. Although the structure of the friction damper is simple, the dynamic behavior of the lubrication damper was not easy to develop a mathematical model. In this paper, several damping models are introduced. Linear damping and quadratic damping had been widely used for the internal damping. Coulomb damping was also used, and a MSTV damping model was recently proposed. Kelvin-Voight damping model was also adopted for a three dimensional elastic damper like bushing. A modified Bouc-Wen model was also proposed to include the hysteretic behavior of an elastomer like bushing in automobiles.

## 2. Introduction

Linear damping : To simulate the effects of combined internal and external structural damping of motion, the simplest model of damping forces is a linear damping, in which the vector of generalized damping forces is  $\mathbf{Q}^d = \mathbf{D}\dot{\mathbf{e}}$ . In this model, a particular form of proportional Rayleigh damping [1] is employed and the damping matrix  $\mathbf{D}$  takes the form  $\mathbf{D} = \alpha \mathbf{M} + \beta \mathbf{K}$  with mass matrix  $\mathbf{M}$  and stiffness matrix  $\mathbf{K}$ . Coefficients  $\alpha$  and  $\beta$  depend on frequencies  $\omega_1$  and  $\omega_2$  as well as on damping ratios  $\zeta_1$  and  $\zeta_2$  in the first two modes of the system. They should be estimated from the experimental data. This approach was successfully used for a simulation of damped large oscillations of beams and showed a reasonable agreement with the experimental results [2].

Quadratic damping : In other cases, for example in the case of large oscillations of a plate, the simple linear model cannot accurately account for the damping effects because of the large cross section of the plate in the direction of motion. Then, a quadratic-in-velocities damping model should be used. It can be represented as  $\mathbf{Q}^d = (\beta_1 + \beta_2 \|\mathbf{v}_c\|) \mathbf{M}\dot{\mathbf{e}}$ , where  $\mathbf{v}_c$  is a characteristic velocity of the element (e.g. the velocity of the center of mass), whereas  $\beta_1$  and  $\beta_2$ , again, are some coefficients obtained from experimental data. This approach was successfully used in reference [3].

Coulomb damping and Stick transition velocity model : A friction damper used in washing machine is shown in Fig. 1. A Coulomb friction model was applied for this damper, then a STV (stick transition velocity) friction model shown was applied. In the STV model, the maximum friction force increases linearly when the velocity is smaller than the STV value. If the value of the STV is zero, the STV model becomes the same as the Coulomb model. Thus, the key factor in the STV model is the magnitude of the STV value, which may depends on the frequency and amplitude of the sponge deformation. As shown in Fig. 2, damping forces in friction damper are dependent on the external forcing frequencies. Thus, to precisely represent the hysteretic behavior, STV values should be adjusted depending on the frequency and amplitude, which is very tedious and difficult to apply in actual modeling. Thus, a MSTV (modified stick transition velocity) model in Fig. 2 was proposed for the friction damper in which the deformation velocity of the sponge is selected for the input variable for the damper.

$$f_{\text{fric}} = \begin{cases} -\text{sign}[\dot{X}(t)] f_{\text{max}} & \text{when } |\dot{X}(t)| \geq |\dot{Y}(t) - \dot{X}(t)| \\ -\frac{f_{\text{max}}}{|\dot{Y}(t) - \dot{X}(t)| + \varepsilon} \dot{X}(t) & \text{when } |\dot{X}(t)| < |\dot{Y}(t) - \dot{X}(t)| \end{cases} \quad (1)$$

where  $\varepsilon$  is a small value to prevent to denominator becomes zero when  $|\dot{Y}(t) - \dot{X}(t)| = 0$ .

Bouc-Wen model : A typical bushing used in automobile is shown in Fig. 3. This type of bushing generally shows a hysteretic behavior depending on frequency and amplitude. Fig. 4 shows forces depending on amplitudes, which shows a hysteretic behavior. To represent this behavior, the hysteretic damping model was developed by adding a nonlinear hysteretic characteristic of the bushing, as shown in Fig. 5. The left-hand side and right-hand side of Fig. 4(a) represent a linear part and a hysteretic part, respectively.



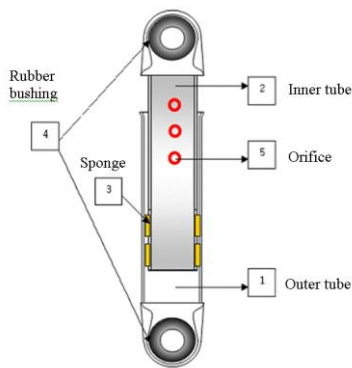


Fig. 1 Friction damper

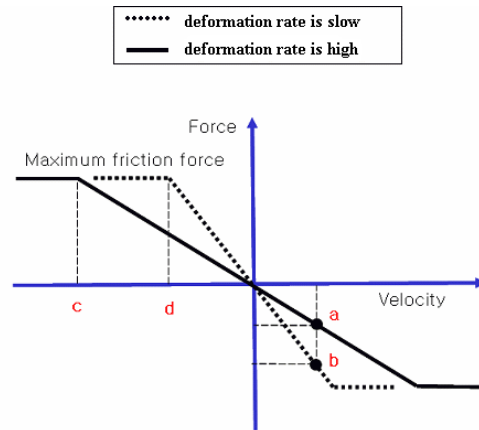


Fig. 2 MSTV model

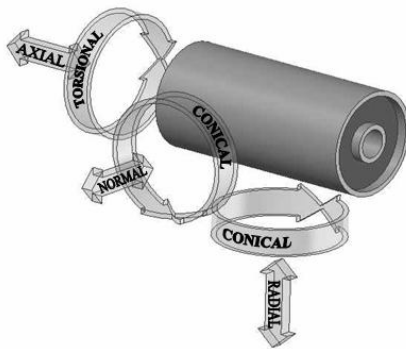


Fig. 3 Bushing deformations

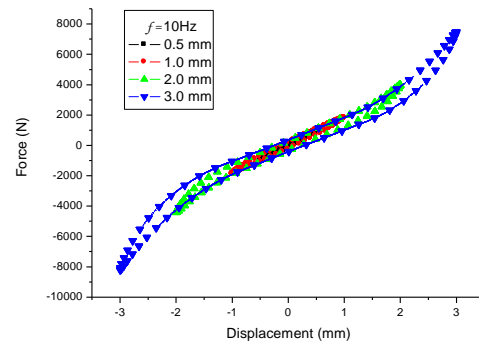


Fig. 4 Hysteretic loop in bushing force

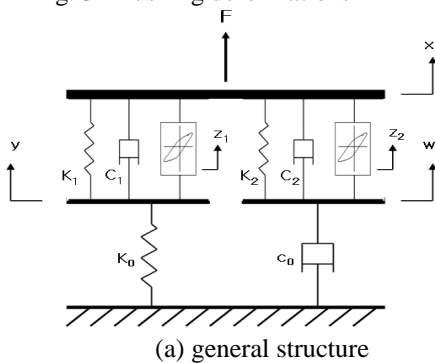


Fig. 5 Modified Bouc-Wen model

### 3. Conclusions

In this paper, several damping models are introduced. For beam or plate, a linear damping or quadratic damping could be applied. For a friction damper, a modified stick transition velocity model is reasonable. And for a bushing in automobiles, a modified Bouc-Wen model is recommended.

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# Spatial Frictional Impact of Rigid and Flexible Multibody Systems

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**Keywords:** impact, impulse momentum equations, friction, multibody systems

## 1. Introduction

Mainly two approaches have been applied for solution of impact problems [1]. An approach that regards the elastic nature of the colliding bodies and definition of the contacting forces as a result of the mutual penetration of the contacting surfaces is widely used but the convergence of the integration procedure could be the braking point. Another approach is based on the Impulse Momentum Equations including the constraint impulses [2]. This approach is successfully applied in practice but also experiences some difficulties. For the plane impact the direction of the tangential velocity and the frictional impulse is constant for the impact time and only their signs (positive or negative) and amplitudes are to be estimated. In case of spatial impact the direction of the sliding velocity is not constant - a problem that is not regarded in the scientific literature.

The paper deals with an algorithm for dynamic analysis of spatial impact of rigid and flexible multibody systems. The impact theory based on Impulse Momentum Equations (IME) and complementary conditions are applied. The hypotheses of Newton and Poisson for the coefficient of restitution, and the nonlinear Coulomb friction are regarded. Numerical procedure for estimation of the sliding velocity is developed. Sticktion and stick-slip processes are analyzed. The transition effect of sliding to sticktion is discussed.

## 2. Definition of spatial impact direction

The assumption that the direction of the sliding velocity during the phenomena impact coincides with the approaching tangential velocity could give inaccurate results. In Fig. 1(a) a falling rigid body is shown. The initial normal  $\mathbf{v}_n$  and tangential  $\mathbf{v}_\tau$  velocities in the contact point, as well as, the visual test of the trace patch as a result of sliding occurred during the impact process are presented (Fig. 1b). It could be observed that the trace does not coincide with the direction of the approaching tangential velocity, which means that this direction does not coincide with the direction of the actual tangential friction velocity during the impact event.

The transformation process of the velocity jump into impulses (or vice versa) is normally expressed [2] by one step cyclic graph. Actually the IME are parametric system of linear equations [3] that depend either on the velocity jump (Newton hypothesis) or on the accumulated impulse (Poisson hypothesis). The IME are:

$$\Delta = \mathbf{M}^{-1} \cdot \mathbf{P}, \quad (1)$$

where  $\mathbf{M}$  is the mass – matrix of the system,  $\Delta$  is the matrix - vector that should be calculated (coordinate velocity jumps and/or unknown impulses), and  $\mathbf{P}$  are the known velocity jumps or accumulated impulses. Since the system configuration is one and the same at the time of impact  $\mathbf{M}$  is assumed constant. The chart of the linear dependence (Eq. 1) between the normal impulse  $\mathbf{p}_n$  and velocity  $\mathbf{v}_n$  for the case of the Newton hypothesis applied for plane impact (simplicity of the drawing) and sliding is presented in Fig. 2 (a). In the figure the chart is discretized and the IME are valid for all its parts  $\Delta \mathbf{v}_n^i$  and the corresponding  $\Delta \mathbf{p}_n^i$ ,  $i = 1, 2, \dots$ ; the superscripts “c” and “r” denote compression and restitution, respectively. In Fig. 2(b) the chart “normal velocity

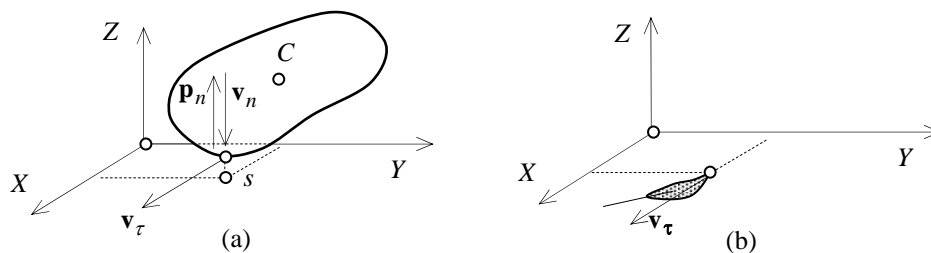


Figure 1. (a) spatial impact of rigid body, and (b) the trace patch of sliding during the impact

– normal impulse” in case of nonlinear value of the friction coefficient is shown. The idea of discretization is extended to the spatial impact. In Fig. 2 (c) sliding during the transformation of the normal velocity is displayed. The normal velocity and its discretization, as well as, the magnitude and direction of the resulting tangential velocity are presented. The main assumption of the method proposed here is that the frictional impulse direction does not coincide with the approaching tangential velocity during the entire stage of the compression and restitution. To find the sliding velocity direction within the impact event it is applied an iterative scheme of discretization of the normal velocity and recalculation of the friction velocity, respectively the frictional impulse, according to the pre-calculated tangential velocity jump.

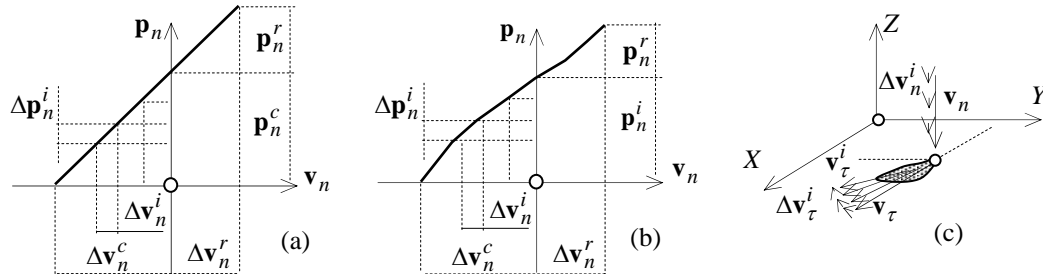


Figure 2. Discretization of the: (a) velocity – impulse diagram; (a) the diagram in case of nonlinear coefficient of friction; (c) sliding velocity

This approach to impact dynamics simulations offers significant advantages. The conditions for stiction or sliding [4] could be very easily verified since the normal velocity is discretized and the first step could be implemented with very small increment. The phenomena sliding – reverse sliding or sliding – stiction are simulated within the iteration procedure. If at certain stage of the calculations the tangential velocity changes its direction this means that some of these phenomena occur and the algorithm switches the procedure to the corresponding branch.

An algorithm is developed that realizes the approach proposed in the paper. Numerical results of spatial impact simulation of rigid and flexible beam are presented in Fig.3. In Fig. 3(b) the trace of the contact point in case of rigid body beam impact is shown. In Fig. 3(c) the coordinates of flexible beam contact point are presented in case of sliding. The case of stiction is presented in Fig. 3(d). Although that no experiments are conducted the approach is the only way to define the tangential velocity for the IME in case of spatial impact and theoretical basis for further investigations.

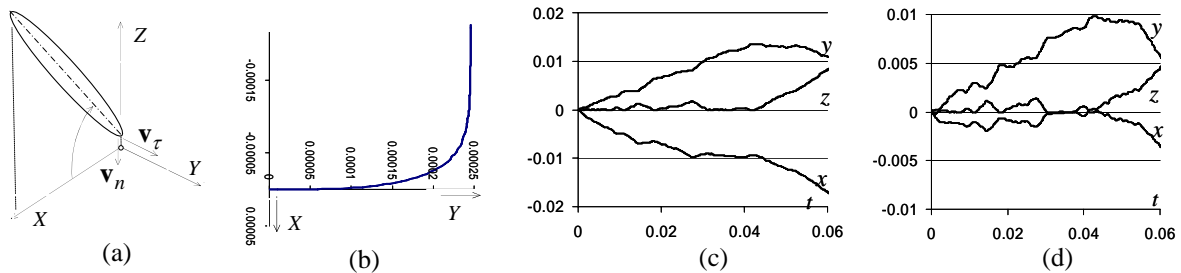


Figure 3. Spatial beam impact: (a) the beam – rigid or flexible; (b) rigid impact – trace of the contact point; flexible impact – coordinates of the contact point in case of (c) sliding, (d) stiction.

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# Computer modeling of electric locomotive as controlled electromechanical system

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**Keywords:** Multibody system, computer modeling, electric locomotive, AC traction drive.

## 1. Introduction

The paper presents observation of the electric locomotive as a controlled electro-mechanical system. The system includes the mechanical part (system of interconnected solids), electric part (energy conversion devices and traction motor drives) and control block (control algorithms and their realization). At the stage of design, it is necessary to study the interrelated electromechanical processes, arising at various regimes of electric locomotive functioning, construct optimal control algorithms. It is relevant to calculate the dynamic processes in the traction electric drive at starting.

The six-axle double-current passenger electric locomotive EP 10, equipped with the asynchronous traction motor drives [1], was used as a prototype. To research the processes at the traction electric drive the complex computer model was constructed, including models of the mechanical part, as well as the contact point "wheel-rail" model; electric part; control system.

## 2. Main body

### 2.1. Modeling of mechanical part

The calculating scheme of electric locomotive mechanical part is shown as a system of solids, comprising a body and three bogies. The bogies are made with two axles of separate drive and transmission of II class. Inclined rods carry out the transmission of the traction and brake efforts from the bogies to the locomotive body. At total, the calculating scheme of the mechanical part comprises of 28 solids and 78 degrees of freedom. The differential equations of movement are set up applying the Newton-Euler formal method, which turned out to be extremely effective at complicated technical systems modeling. The differential equations of movement are set up automatically using computer algebra aids. Consequently, there are practically no limitations for the bodies' quantity and degrees of freedom number, it enabled to achieve more complete and accurate mechanical processes modeling.

Modeling of dynamic processes at the contact point "wheel-rail" is of great importance when studying such unsteady regimes as wheel slipping and skidding, etc. To calculate the traction effort Kalker theory was used, the contact "wheel-rail" is considered resilient, and the traction effort is calculated taking account of deformation and partial wheel and rail sliding. On this basis, the algorithm for calculation of the traction effort under unsteady rolling conditions was developed, which enables to create animated pictures of initiation and alteration of traction efforts, distributed along the contact spot, as well as at locomotive at starting.

### 2.2. Modeling of electrical part.

The energy conversion system is designed in the mode of electric circuit with intense parameters. The energy conversion system comprises of transformer, inverting converters and independent voltage inverters, used as phase frequency and number converters for traction motor drive feeding. To calculate them, electric circuit dynamic synthesis method is being used, which is the discrete models method adaptation. It enables to set up the equation of the electric circuit condition automatically, and discard difficulty limitations of the given system.

The conversion of electromagnetic energy into mechanical is carried out in the asynchronous traction motor drive. Motor drive magnetic condition determination is carried out by the field theory methods, applying the final elements method. In the mathematical model the saturation and teeth of the active layer of the stator and rotor, displacement of current in the stator windings and rotor windings are taken into consideration. This enables to carry out investigations at various regimes, including stopping (starting) and running low speed. The computer model was set up applying to the traction motor drive with the hour regime power 1200 kW. The animated picture of the power lines and electromagnetic field intensity was created.

### **2.3. The control system.**

A number of specific demands are made to the asynchronous traction electric drive control. It is stipulated by peculiarities of its application, specifically, by the changing conditions at the “wheel-rail” contact point that leads to the structural unsteadiness of the system. ATM moment regulation should be carried out without oscillations and extra regulation, and its rapidness should be sufficient for prevention of wheel slipping.

The analysis has shown asynchronous traction drive most rational control system is the vector regulation control system. The control system sustains the given magnetic flux linkage of the rotor by changing the stator current magnitude. On its basis, the two-channel, four-circuit regulation system of the asynchronous traction electric drive with the flux linkage and moment independent control was created.

The elaborated electric locomotive computer model enabled to fulfill complex evaluation of the control system functioning principles and algorithms effect on current conversion devices and asynchronous traction motor drives operation. It conducted to analyze electromechanical processes at various operation modes, such as traction, curve passing, etc. Of special interest are these processes research results at the starting regime, aggravated coupling conditions, urgent braking, etc.

Calculation results have shown that joint processes in the electric locomotive electromechanical system have complex oscillatory character. At inappropriate control algorithms, due to resonance frequencies, relevant dynamic loads in the mechanical part are possible. Excessive wheel slipping and skidding of one or several most unloaded wheel sets should be immediately revealed and suppressed; corresponding procedures are included into the control algorithm. As a criterion, characterizing wheel slipping arising danger, the coupling reserve value is taken (difference between potentially possible coupling power and current traction power value).

### **3. Conclusions**

Applying given software package, joint electromechanical processes, arising at various control variants, were investigated. The results are used for calculating dynamical loads in the mechanical part, improving locomotive automatic control systems and creating new, more effective systems for wheel set slipping and skidding suppression.

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